- $<$ means less than
- $>$ means greater than
- $\leqslant$ means less than or equal to
- $\geqslant$ means greater than or equal to
- $\neq$ means *not* equal to

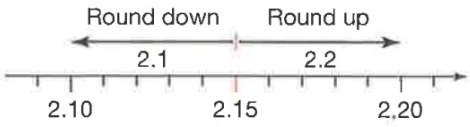
EXAMPLE

Place the correct symbol $<$, $>$ or $=$ between the numbers in each pair.

a 5.07 5.7 **b** 397 379 **c** −10 5 **d** −19 −24 **e** $\frac{3}{2}$ 1.5

a 5.07 < 5.7 **b** 397 > 379 **c** −10 < 5 **d** −19 > −24 **e** $\frac{3}{2} = 1.5$

- To round a number look at the next, smaller, digit
  - next digit = 0, 1, 2, 3 or 4 round down
  - next digit = 5, 6, 7, 8 or 9 round up.



EXAMPLE

Round 72456.0374 to the nearest

a ten **b** hundred **c** thousand **d** tenth **e** hundredth **f** thousandth.

a 72460 **b** 72500 **c** 72000 **d** 72456.0 (1 dp) **e** 72456.04 (2 dp) **f** 72456.037 (3 dp)

- When rounding to **significant figures**, count from the first non-zero digit.

EXAMPLE

a Round these numbers to 2 dp.

i 34.567 **ii** 3.887 126 **iii** 215.587 54

b Round these numbers to 2 sf.

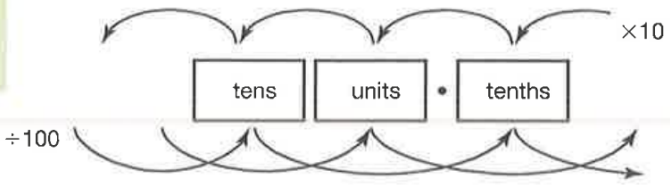
i 39.54 **ii** 217 **iii** 0.000 455 **iv** 12 019 **v** 25.505

dp means 'decimal places' and **sf** means 'significant figures'.

a **i** 34.57 **ii** 3.89 **iii** 215.59

b **i** 40 **ii** 220 **iii** 0.000 46 **iv** 12 000 **v** 26

- Multiplying or dividing a number by a power of 10 changes the place value of each digit.



Multiplying by 10 moves the digits one place to the left. Dividing by 100 moves the digits two places to the right.

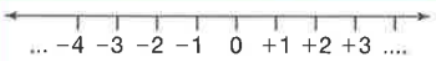
EXAMPLE

Work out **a** 3.72 ÷ 100 **b** 0.0349 × 10 000 **c** 17.3 ÷ 1000

a Move the digits 2 places right. 0.0372

b Move the digits 4 places left. 349

c Move the digits 3 places right. 0.0173

Language	Meaning	Example
Place value	The value of a digit according to its position in a number.	123.4 2 means 2 tens = 20 4 means 4 tenths = $\frac{4}{10}$
Rounding	Expressing a number to a given degree of accuracy.	103.67 = 103.7 (1 dp) = 100 (1 sf) 0.0055 = 0.0 (1 dp) = 0.006 (1 sf)
Decimal places	The number of digits after the decimal point.	
Significant figures	The number of digits after the first non-zero digit.	
Directed number	A number with a plus or minus sign in front of the number	
Negative	A number that is less than zero	3 − 6 = −3
Estimate	An approximate calculation or a judgement of a quantity.	Estimate 68.89 × 21.1 ≈ 70 × 20 = 1400 Exact = 1453.579
Partitioning	Splitting a larger number into smaller numbers which add up to the original number.	85 + 25.6 = 85 + (15 + 10.6) = 100 + 10.6 = 110.6
Compensation	Replacing a number by a simpler approximate value and a correction.	158 − 18.9 = 158 − (20 − 1.1) = (158 − 20) + 1.1 = 139.1
Operations	Rules for processing numbers.	Addition, subtraction, multiplication and division.
Order of operations	The order in which operations have to be carried out to give the correct answer to a calculation.	2 + 4 × 3 − 1 = 2 + 12 − 1 = 13 (2 + 4) × 3 − 1 = 6 × 3 − 1 = 18 − 1 = 17 (2 + 4) × (3 − 1) = 6 × 2 = 12 2 + 4 × (3 − 1) = 2 + 4 × 2 = 2 + 8 = 10
BIDMAS	An acronym for the correct order of operations: **B**rackets, **I**ndices (or powers), **D**ivision or **M**ultiplication, **A**ddition or **S**ubtraction.	

Objectives

By the end of this chapter, you will have learned how to ...

- Perform calculations involving roots and indices, including negative and fractional indices.
- Perform exact calculations involving fractions, surds and π.
- Work with numbers in standard form.

Calculating with roots and indices

- Indices are defined for fractional and negative powers.

$x^{\frac{1}{n}} = \sqrt[n]{x}$ $\quad x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ $\quad x^{-n} = \frac{1}{x^n}$ $\quad x^0 = 1$

- The same index laws we use for positive whole number powers also apply to fractional and negative powers.

$x^a \times x^b = x^{a+b}$ $\quad x^a \div x^b = x^{a-b}$ $\quad (x^a)^b = x^{a \times b}$

$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$

E.g.

1. Arrange these numbers in ascending order.

$27^{-\frac{2}{3}}$ $\quad 9^{\frac{3}{2}}$ $\quad 16^{\frac{3}{4}}$ $\quad \left(\frac{1}{2}\right)^{-4}$ $\quad \left(\frac{1}{8}\right)^{-\frac{5}{3}}$ $\quad 25^0$

$\frac{1}{9}$ $\quad 27$ $\quad 8$ $\quad 16$ $\quad 32$ $\quad 1$

2. Simplify these expressions.

a $2^{-\frac{3}{2}} \times 2^{\frac{1}{4}}$ $\quad$ **b** $2^{-\frac{1}{6}} \div 2^{-\frac{1}{3}}$

c $(2^{\frac{1}{8}} \div 2^{-\frac{1}{2}})^3$ $\quad$ **d** $2^{\frac{3}{2}} \times 2^{-2} \div 2^{\frac{1}{4}}$

e $(2^{\frac{5}{4}} \times 2^{-1})^{-\frac{3}{2}}$ $\quad$ **f** $(2^3 \div 2^{-\frac{1}{2}})^{-2} \times (2^{\frac{3}{2}} \div 2^{-1})$

3. Solve these equations.

a $(2^{-3x} \div 2^{-2})^2 = 2^{10}$

b $\frac{2^{3x} \times 2^4}{2^3 \times 2^{4x}} = \frac{1}{2^2}$

c $(2^{4+x} \div 2^{2+2x})^2 = (2^{3x} \times 2^{x-2})^{-1}$

d $2^{\frac{3}{2}} \times 2^x \div 2^{\frac{2}{3}} = 2$

e $(2^{\frac{1}{2}} \times 2^x)^{\frac{1}{2}} = \frac{1}{(2^{\frac{1}{3}})^3}$

Exact Calculations

Question type: Evaulate these answers exactly.

- To find an exact answer, do *not* use decimals. Instead use fractions in their lowest terms, simplified surds and multiples of π, as appropriate, throughout the calculation.
- A surd is in its simplest form when the smallest possible integer appears inside the square root.

E.g.

Calculate this expression

a exactly

b to 3 dp.

$$\frac{7+\sqrt{20}}{4}-\frac{3+\sqrt{5}}{5}$$

Evaluate these calculations exactly.

a $\frac{2}{5}\times\left(\frac{3}{4}+\frac{2}{3}\right)$ **b** $\left(\frac{2}{3}+\frac{4}{5}\right)\div\left(\frac{2}{5}+\frac{3}{7}\right)$

c $\frac{5}{6}\div\frac{3^2+4^2}{10}$ **d** $\left(\frac{3}{7}\right)^2\times\left(\frac{4}{5}-\frac{1}{7}\right)$

e $\left(\frac{1}{2}+\frac{5}{9}\right)^2+\left(\frac{2}{3}\right)^3$

Standard Form

- A number in standard form is written $A\times10^n$, where n is a positive or negative integer and $1\leqslant A<10$.

E.g.

1.

Simplify these expressions.

a $(3.6 \times 10^5) \div (1.2 \times 10^3) = 3 \times 10^2$

b $(5.4 \times 10^4) \times (2 \times 10^{-3}) = 10.8 \times 10^1 = 1.08 \times 10^2$

2.

A bumblebee has mass 5.2×10^{-5} kg.
An adult man has mass 70 kg.
A bumblebee can carry 75% of its mass.
How many bumblebees would it take to lift the man?
Give your answer in standard form to 3 sf.

$70 \div 5.2 \times 10^{-5} \times \frac{3}{4}$

$= 1794871.795$

$= 1796000$

$1.79 \times 10^6 = 1790000$

$A \times 10^n$

Practice:

9 **a** The Wright Brothers Flyer I, the world's first aircraft, had a mass of 3.4×10^2 kg.
The Saturn V Rocket had a mass of 2.96×10^6 kg.
How many times heavier is a Saturn V than a Flyer I? [2]

b Flyer I attained a speed of 3.04×10^0 ms^{-1} on its first flight and the supersonic airliner, Concorde, attained a speed of 2.179×10^3 kmh^{-1}.
How much faster was Concorde than Flyer I? [2]

c There are approximately 4.336×10^9 stars in our galaxy and about 5.776×10^3 stars visible to the naked eye. What fraction of the galaxy can we see?
Write your fraction in the form $\frac{1}{x}$. [2]

d A triathlon has 3 stages. The largest triathlon has a 3.8×10^0 km swim, a 1.8×10^2 km cycle ride and a $0.421\,95 \times 10^2$ km run.
How far is the race in total?
Give your answer as an ordinary number to 3 sf and in standard form. [3]

$1 \div 0.750692 \times 10^6 = 1.75069...$

$5.776 \times 10^3 \div 4.336 \times 10^9$