**G8**

| Core content | Extension content |
|---|---|
| identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference including: tangent, arc, sector and segment | apply the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results |

**Notes:** including angle subtended by an arc at the centre is equal to twice the angle subtended at any point on the circumference, angle subtended at the circumference by a semicircle is 90°, angles in the same segment are equal, opposite angles in a cyclic quadrilateral sum to 180°, tangent at any point on a circle is perpendicular to the radius at that point, tangents from an external point are equal in length, the perpendicular from the centre to a chord bisects the chord, alternate segment theorem.

**G16**

| Core content | Extension content |
|---|---|
| know and use the formulae:<br>circumference of a circle = $2\pi r = \pi d$<br>area of a circle = $\pi r^2$<br>calculate perimeters and areas of 2D shapes, including composite shapes | surface area and volume of spheres, pyramids, cones and composite solids including composite shapes and frustums of pyramids and cones |

**Notes:** solutions in terms of $\pi$ may be asked for.

**G18**

| Core content | Extension content |
|---|---|
| | calculate arc lengths, angles and areas of sectors of circles |

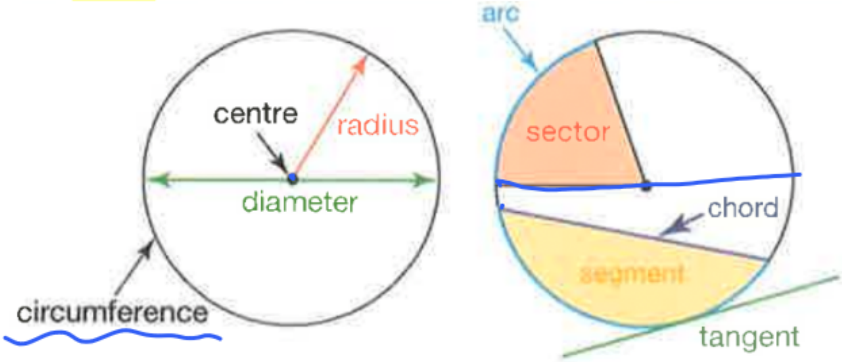
**Study Goals:**

- Circle formulae
  - Circumference and areas of a circle
  - Arc lengths
  - Areas of sectors / segments
- 8 Circle theorems (use them to prove related results)
- Constructions and loci -> compass
  - Draw **perpendicular bisector**
  - Draw **angle bisector**

**Vocabularies**

| Name | Translate | Name | Translate |
| --- | --- | --- | --- |
| circumference | 周长 | locus<br>( pl. : loci) | 轨迹 |
| Radius<br>( pl. : radii ) | 半径 | Angle bisector | 角平分线 |
| diameter | 直径 | Perpendicular bisector | 垂直平分线 |
| Arc length | 弧长 | subtended angle | 夹角 |
| sector | 扇形 | Cyclic quadrilateral | 圆内接四边形 |
| segment | 片段 | Angle at the circumference /center | 圆周角 |
| chord | 弦 | Perimeter<br>/pəˈrɪmɪtə(r)/ | 外缘；周长 |
| tangent | 切线 | | |

## 1. **Terms**

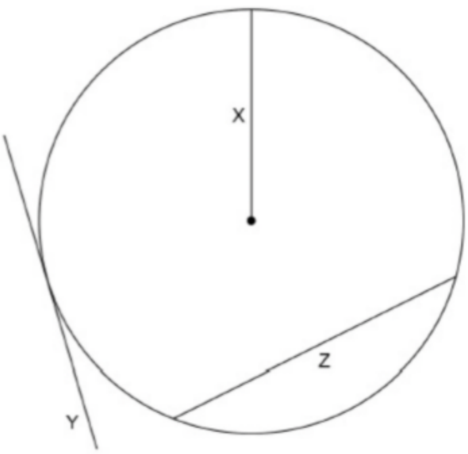



line segment 线段

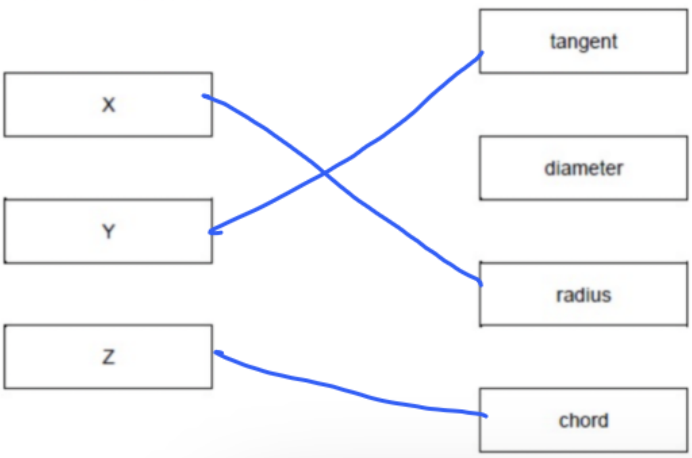
- Arc: A part of the curve along the perimeter of a circle.
- Sector: A portion of a circle resembling a 'slice of pizza'.
- A chord of a circle is a line segment (线段) that joins any two points on its circumference; A diameter is the longest chord possible.
- A segment is a region bounded by a chord and an arc of the circle.
- Tangent: a straight line which touches the circle at only one point (so it does not cross the circle - it just touches it).

Q22N

Here is a circle and lines X, Y and Z.

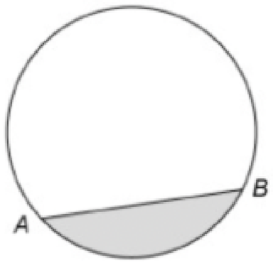



Match each line to its correct name.



Q21N

*A* and *B* are two points on a circle.




Complete each statement using a word from this list.

| arc | sector | chord | segment | tangent |
|---|---|---|---|---|

The shaded area is a Segment

The straight line *AB* is a Chord

## 2. Circle formulae

### (1) Circumference & Area

RECAP

- Diameter of a circle = 2 × radius : $d=2\cdot r$
- Circumference of a circle, $C = \pi d$ or $2\pi r$

  Rearranging gives $d = \frac{C}{\pi}$ or $r = \frac{C}{2\pi}$
- Area inside a circle, $A = \pi r^2$

  Rearranging gives $\boxed{r = \sqrt{\frac{A}{\pi}}}$

C; $d=\frac{C}{\pi}$

### (2) Arc length & Area of sector

$d=2r$

- Arc length $s = \frac{\theta}{360°} \times 2\pi r$ or $\frac{\theta}{360°} \times \pi d$
- Area of sector $A = \frac{\theta}{360°} \times \pi r^2$ Area of a circle

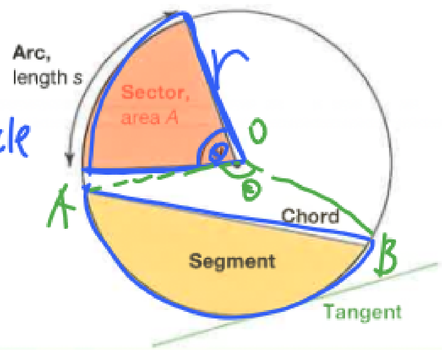
周长

The perimeter of a sector consists of $\boxed{\text{an arc}}$ and two radii.

The perimeter of a segment consists of an arc and a chord.

To find the area of a segment, you will need to use the area of a sector and subtract the area of a triangle.

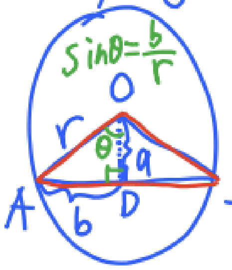
Ch18




Chord length=

① Pythagoras Theorem $a^2+b^2=r^2$ $b^2=r^2-a^2 \ (b>0)$

chord + AB



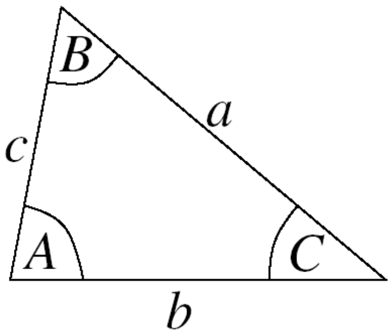

$AB=2\sqrt{r^2-a^2}$

$b=r\cdot\sin\theta$

$AB=2\cdot r\cdot\sin\theta$

② $\sin\theta=\frac{c}{b}$. $\cos\theta=\frac{a}{b}$


$\tan\theta=\frac{c}{a}$.




Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(for finding sides) (for finding angles)

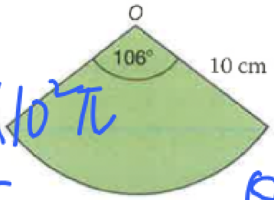
Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad \text{or} \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

(for finding sides) (for finding angles)

**Example:**

**a** Find the area of this sector.

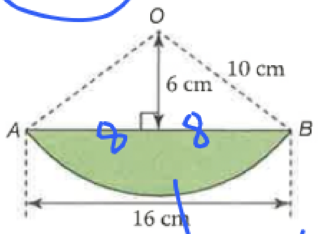



$\frac{106^\circ}{360^\circ} \times 10^2\pi$

$= \frac{265}{9}\pi\ cm^2$

$\frac{\theta}{360^\circ} \times \pi r^2$

**b** Triangle *OAB* is removed from the sector. Find the area of the remaining segment.




$8 \times 2 \times 6 \div 2$

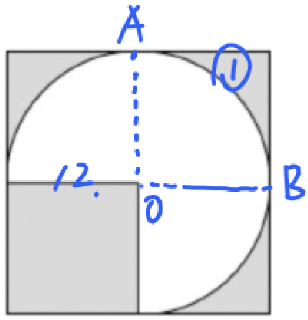
$= 48 cm^2$

$\frac{265}{9}\pi - 48\ cm^2$

**Practice:**

**Q22N**

Three quarters of a circle just fits inside a square.




Sector OAB $= \frac{90^\circ}{360^\circ} \times \pi \times 144$

The circle has radius 12 cm

Work out the shaded area.

**Q20N-1**

A design is made by joining right-angled triangle *OCB* to the sector of a circle *AOB*.

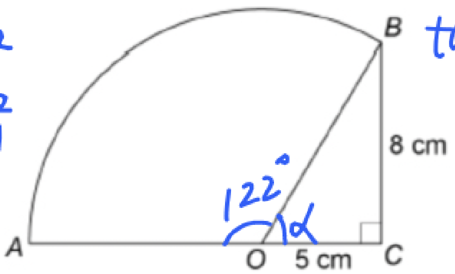
*AOC* is a straight line.

*OC* = 5 cm and *CB* = 8 cm

OAB = 94.75 $cm^2$

△OCB = 20 $cm^2$

114.75 $cm^2$




$\tan\alpha = \frac{8}{5}$

$\alpha = \tan^{-1}\left(\frac{8}{5}\right) = 58^\circ$

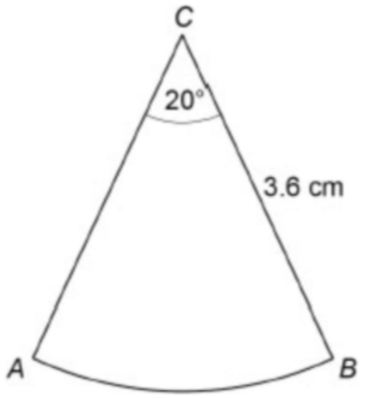
Not drawn accurately

Work out the total area of the design.

The diagram shows an earring made from wire.

*AB* is an arc of a circle, centre *C*.

Not drawn accurately




$CA + CB + \overset{\frown}{AB} = 8.4566$

$7.2$ $\quad \frac{\theta}{360^\circ} \times 2\pi r = \frac{2}{5}\pi$

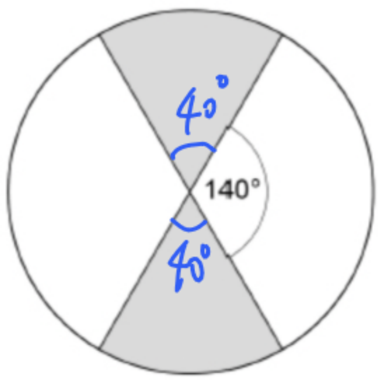
Work out the **total** length of wire used to make the earring.

Give your answer as a decimal.

**Q19N**

A circle has radius 15 cm

Two diameters divide the circle into four sectors as shown.




Work out the total shaded area.

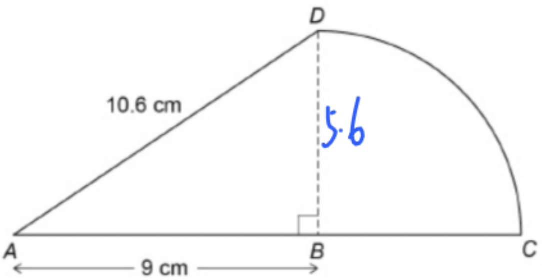
**Q18N**

*ABD* is a right-angled triangle.

*BCD* is a quarter circle, radius *BD*.

$$\frac{90^\circ}{360^\circ} \times \pi \times 5.6^2 = \frac{196}{25}\pi$$

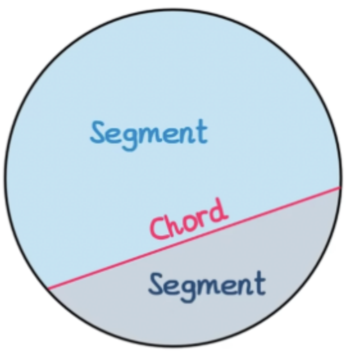
$(cm^2)$




Work out the area of $\boxed{ABCD.}$ 49.83

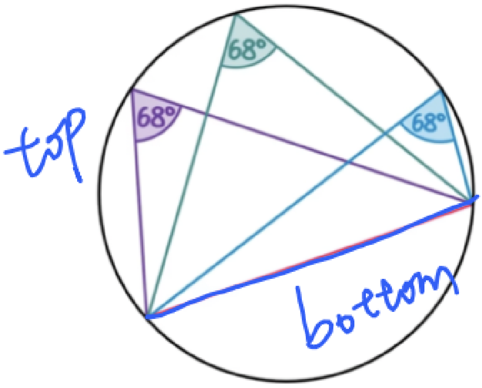
## ★3. Circle theorems

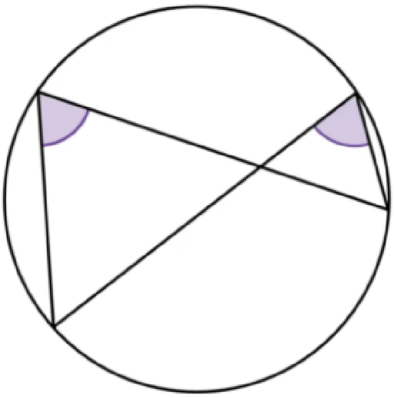
### 3.1 Angles in a circle



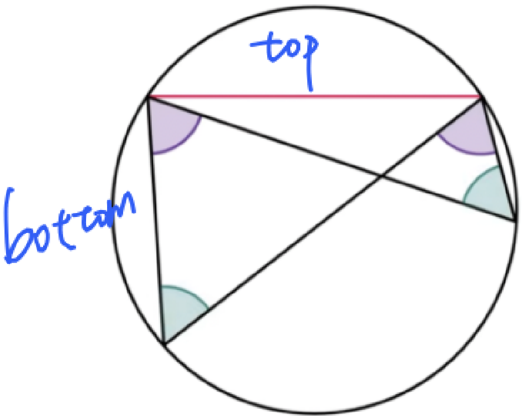
**(1) Theorem 1: Angles in the same segment are equal.**

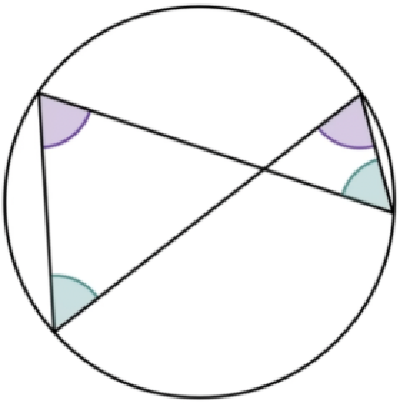
if we use this chord that we've drawn to create an angle but keep that angle within the same segment, all of those angles will be the same size



sometimes this theorem is drawn without the red chord
the property still holds



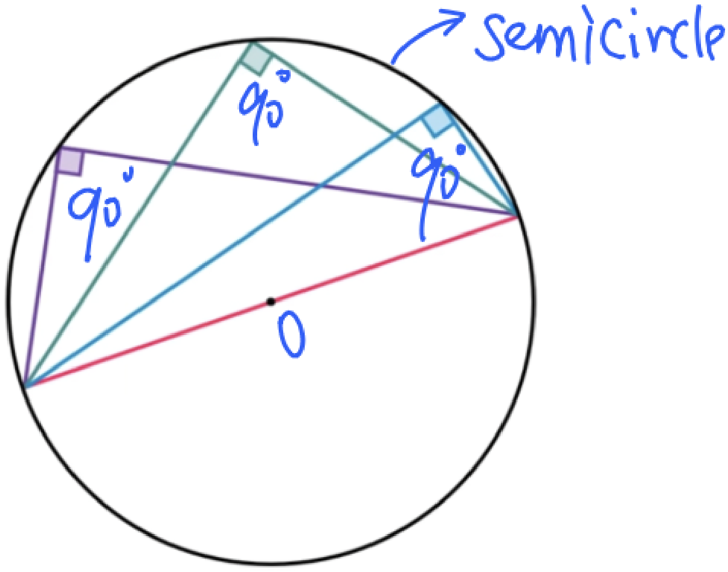
bow tie

you can also apply this theorem by drawing a chord

### (2) Theorem 2: The angle in a semicircle is 90°

- draw a chord that goes straight through the center, this is known as a **diameter.**
- And the two segments are called **semicircle** (because that diameter splits the circle in half)

**From theorem 1, if the segment happens to be the diameter, then angles=90.**

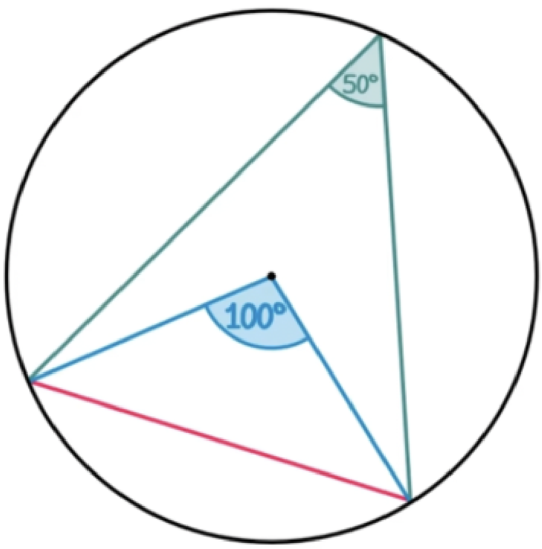



### (3) Theorem 3: The angle at the center is twice the angle at the circumference.

Draw a chord once again and draw two angles:
one at the circumference, and one at the center **but using the same chord**

圆周角

**Q:** Angle at the circumference? Angle at the center?




**In reverse:** the angle at the circumference is half of the angle at the center

(4) Theorem 4: The opposite angles in a cyclic quadrilateral add to 180°

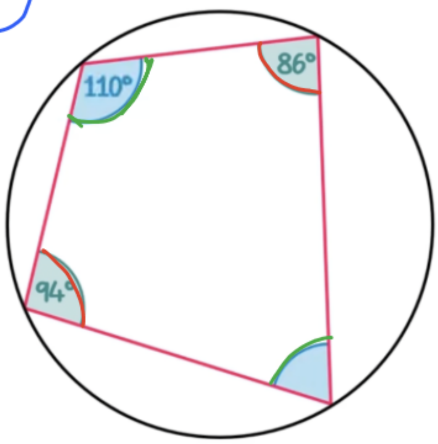
$180° \times (n-2)$

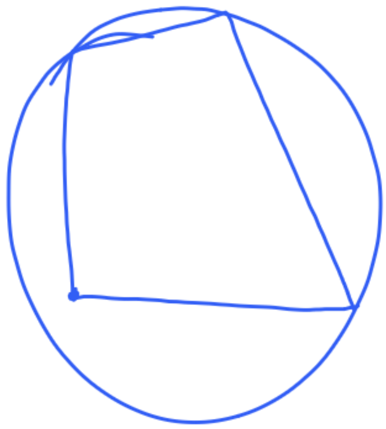
$n = 4$

360°

**Cyclic quadrilateral （圆内接四边形）：**

**all four corners** of the quadrilateral touches the circumference of the circle



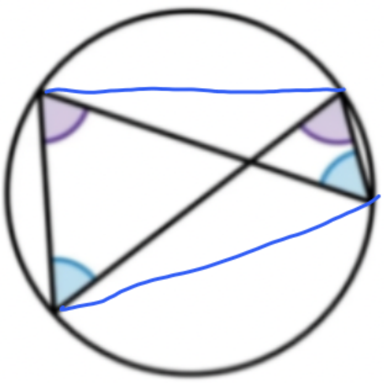
X

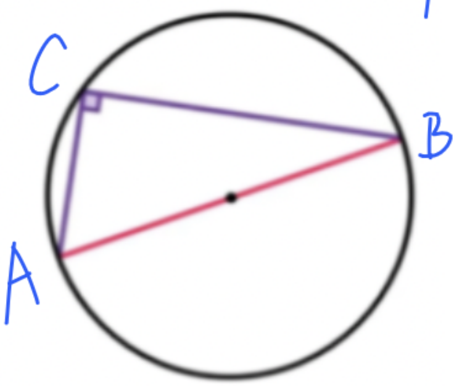
**Quick review:**

prove AB diameter

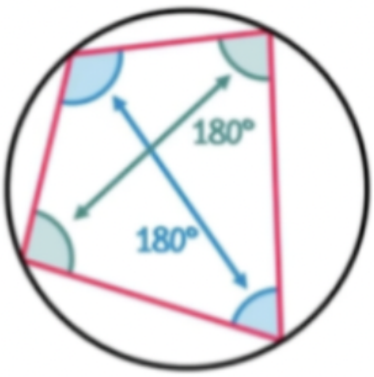
//

prove ∠ACB = 90°






Theorem 1: Angles in the same segment are equal.
Theorem 2: The angle in a semicircle is 90°


Theorem 3: The angle at the center is twice the angle at the circumference.
Theorem 4: The opposite angles in a cyclic quadrilateral add to 180°

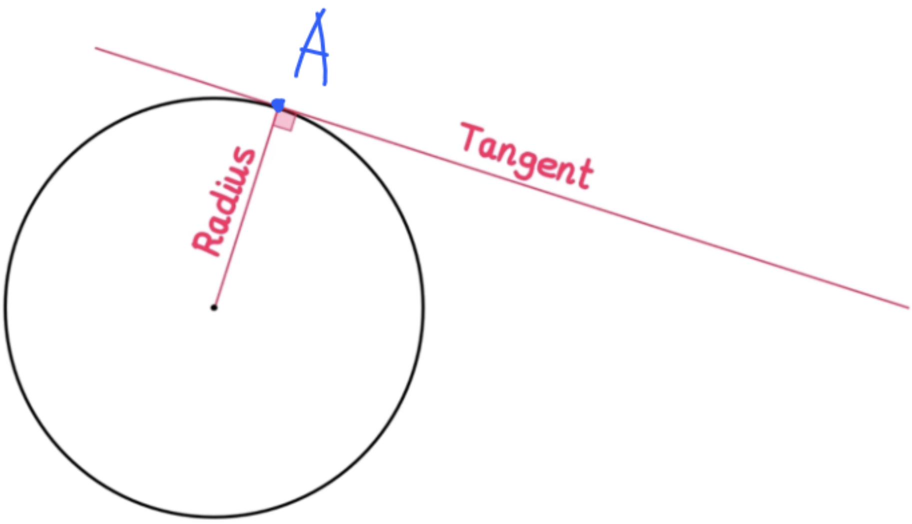
## 3.2 radii, tangents and chords

**(1) Theorem 5:**

**The tangent at any point on a circle is perpendicular to the radius at that point.**

Draw a **tangent** to the circle
(a tangent is a straight line that **touches** the circle in one place)

Draw a radius to the point where the tangent also touches the circle

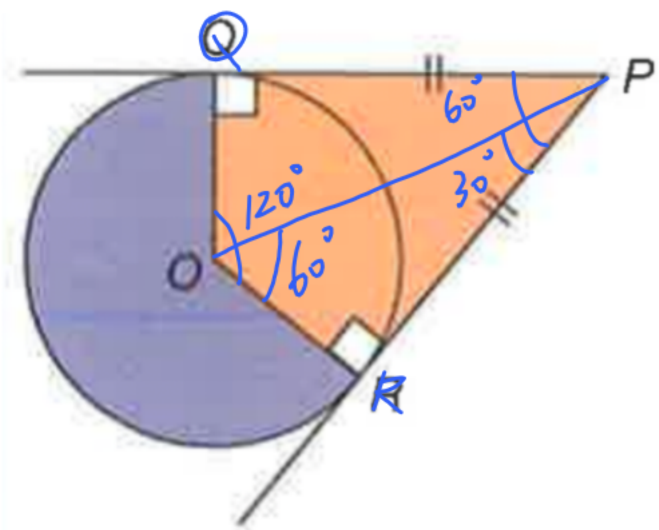



a tangent meets a radius at 90 degrees

**(2) Theorem 6:**

**Tangents from an external same point are equal in length.**

if we create a point at the end of this tangent on the right hand side and call that P
and then if we draw a second tangent from P to the circle

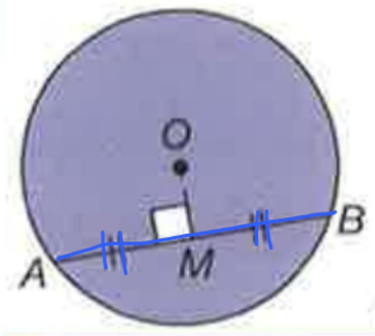



$OR \perp RP$

$PQ = PR$

1- Measure the distance: $\boldsymbol{PQ = PR}$
2- Measure angles: Join OP, and OP is the **angle bisector**

**(3) Theorem 7: The perpendicular from the center to a chord bisects the chord.**

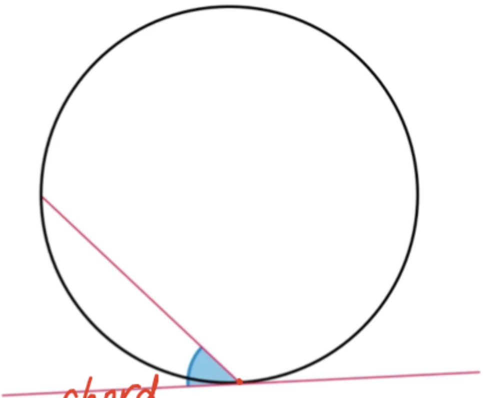



AM=BM

**(4) Theorem 8: Alternate segment theorem**

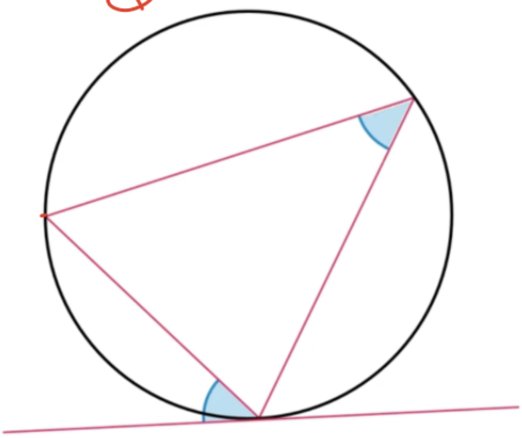
1- draw a tangent to the circle
then draw a chord from the point where the tangent touches the circle

Subtented angle 夹角



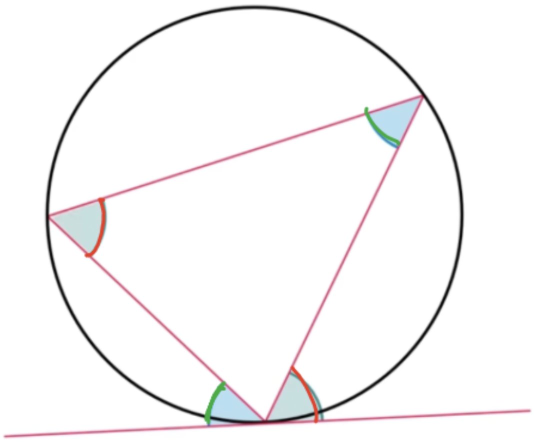
chord

2- use the cord to create an angle at the circumference but not in the same segment



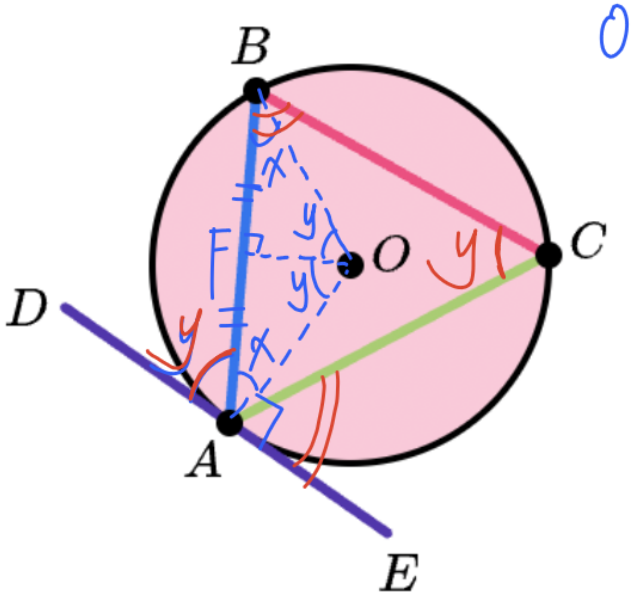
**it turns out that the angle that the tangent makes with the chord is equal to this angle that the chord makes at the circumference**

On the other side:



The angle that lies between a tangent and a chord is equal to the angle subtended by the same chord in the alternate segment. 切线与弦之间的夹角等于同一弦在另一条线段上的夹角

**Proof:**




$OA \perp DE \Rightarrow \angle OAD = 90^\circ$

$\underbrace{x + y = 90^\circ}$ $\qquad$ $\Downarrow$ $\qquad$ $\underbrace{x + \angle BAD = 90^\circ}$

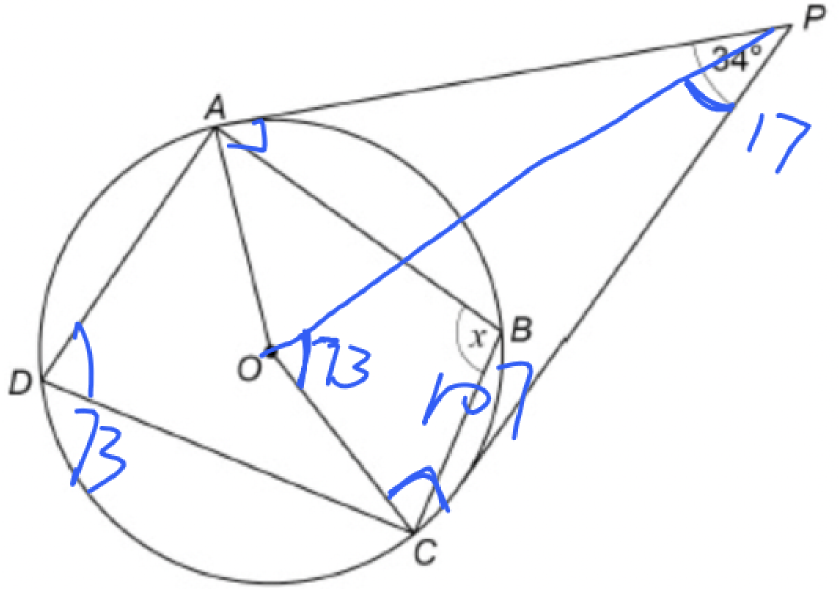
$y = \angle BAD$

**Practice:**

**Q22N-5**

*A*, *B*, *C* and *D* are points on a circle, centre O.

*PA* and *PC* are tangents.




Work out the size of angle $x$.