1. Linear Sequences

- **definition**

- Linear sequences are sequences in which the differences between terms are a constant. This is known as the 'first difference'.

Linear sequences can be generated and described using

- a **'term-to-term' rule**
- a **'position-to-term'** rule

For the sequence
5, 8, 11, 14, 17...
'add 3' 3 3 3 → 1^{st} diff
***n*th term** $= 3n + 2$
0 0 0
arithmetic

EXAMPLE

Find the first five terms of the following sequences using the given term-to-term rules.

a First term 20 Rule Subtract 7 **b** First term 4 Rule Add 6

a 20, 13, 6, −1, −8
−7 −7 −7 −7

b 4, 10, 16, 22, 28
+6 +6 +6 +6

EXAMPLE

The *n*th term of a sequence is $5n - 2$.

a Find the first five terms of the sequence **b** Find the 50th term.

'$5n$' means '$5 \times n$'

a 3, 8, 13, 18, 23 n=1 ~ n=5
1st term ($n = 1$): $(5 \times 1) - 2 = 3$
2nd term ($n = 2$): $(5 \times 2) - 2 = 8$
3rd term ($n = 3$): $(5 \times 3) - 2 = 13$
4th term ($n = 4$): $(5 \times 4) - 2 = 18$
5th term ($n = 5$): $(5 \times 5) - 2 = 23$

b 50th term $= (5 \times 50) - 2 = 248$
n=50

- How to describe a linear sequence using the nth term

HOW TO

To describe a linear sequence using the *n*th term
1. Find the *constant* difference between terms.
2. This difference is the first part of the *n*th term.
3. Add or subtract a constant to adjust the expression for the *n*th term.

2, 6, 10, 14

4n±2

E.g.

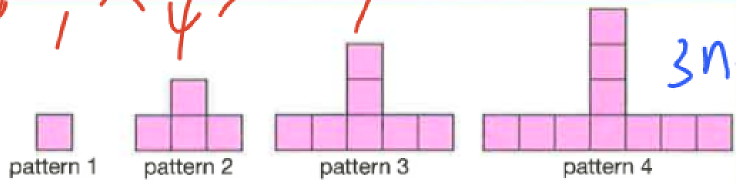
4 4 4 4
2, 6, 10, 14, 18, ...
differences all = +4
nth term rule = $4n \pm \square$
Compare $4n$: 4, 8, 12, 16, 20
2, 6, 10, 14, 18 −2
Sequence = $4n - 2$

	n=1	n=2	n=3
4n	4	8	12
Sequence	2	6	10
+2	2	2	2

Kia says

'there will be 30 squares in the 12th diagram because there are 10 squares in the 4th diagram.'

$n=12.\ 3n-2=34$



Do you agree with Kia? Explain your reasoning.

3n−2 | 3 6 9 12
1 4 7 10

disagree $12\times3-2 = 36-2=34$

$4\times3-2 = 10$

1 2 3 4
1 4 7 10
3 3 3

2. Quadratics Sequences

- Quadratic sequences are sequences in which
  - the second differences between terms are constant
  - the *n*th term rule contains an n^2 term and no higher powers of *n*.
  - The coefficient of n^2 is half the constant second difference.

e.g.

For the sequence
- 2, 5, 10, 17, 26...
  'add the odd numbers starting with 3.'
- *n*th term = $n^2 + 1$

5, 11, 21, 35, 53, ...

6 10 14
4 4

3, 8, 15, 24, 35, ...

5 7 9 11
2 2 2

- **How to describe a quadratic sequence using nth term**

HOW TO

To describe a quadratic sequence using the *n*th term

1. Find the first difference between each term.
2. Find the difference between the first differences: the sequence is quadratic if the second difference is *constant*.
3. The coefficient of n^2 is half the value of the second difference.
4. Add a linear sequence to adjust the expression for the *n*th term.

Example:

Find the nth term of this sequence

4 7 14 25 40

$\frac{4}{2}n^2 + \_n + \_$

①

4 → 7 → 14 → 25 → 40

+3 +7 +11 +15 ✗

+4 +4 4 ✓

Answer: $2n^2$

Step 1: +4 ÷ 2

Sequence	4	7	14	25	40
$-\ 2n^2$	2	8	18	32	50
$-3n + 5$	2	-1	-4	-7	-10

-3 -3 -3 -3

$2n^2 + \underline{-3}\,n + \underline{5}$

$n=1$ $-1+5$ 4

$n=2$ $2+5$ 7

E.g.

Find the *n*th term for these quadratic sequences.

a 5, 11, 21, 35, 53, …

6 10 14

4 4

5	11	21	35
2	8	18	32
3	3	3	3

$2n^2 + 0n + 3$

$2n^2+3$

b 3, 8, 15, 24, 35, …

5 7 9 11

2 2 2

	3	8	15	24	35
n^2	1	4	9	16	25
	2	4	6	8	10

2 2 2 2

n^2+2n

3 8 15

$\underline{1}n^2 + \underline{2}n + \underline{0}$

3. Special Sequences

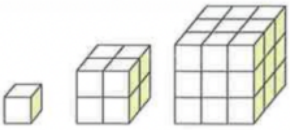
- Square, cube and triangular numbers are associated with geometric patterns.

Square numbers

1, 4, 9, 16, 25, ...

Cube numbers

1, 8, 27, 64, 125, ...



Triangular numbers

1, 3, 6, 10, 15, ...

2 3 4 5

1 1 1

$\frac{1}{2}n^2 + \frac{1}{2}n + 0$

$\frac{n(n+1)}{2}$

1 3 6 10 15

The n-th term of triangular numbers:

$$\frac{n(n+1)}{2}$$

$\frac{1}{2}n^2$ $\frac{1}{2}$ 2 $\frac{9}{2}$ 8

$\frac{1}{2}$ 1 $\frac{3}{2}$ 2

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

- **Arithmetic** (linear) progressions have a constant difference between terms. $T(n+1) - T(n) = d$

Which of these sequences are arithmetic progressions?

a 5, 8, 11, 14, ... **b** 1, −2, 4, −8, ... **c** $3, 3\sqrt{3}, 9, 9\sqrt{3}, \ldots$ **d** 7, 3, −1, −5, ...

- In a **quadratic** sequence the differences between terms form an arithmetic sequence; the second differences are constant.

3 3 3 3   −3 6 −12 $3\sqrt{3}-3$ −4 −4 −4 = d

$a_n = a_1 + (n-1)d$ X geometric   → −4n+11

$5 + 3(n-1) = 3n+2$

$a_n = 7 + (n-1)(-4) = -4n + 11$