OXFORD AQA
INTERNATIONAL QUALIFICATIONS

Center Number: 9 6 2 4 2 Candidate Number: 1 5 7 6 8

Surname: Shi

Forename(s): Hongbin Candidate Signature: 石洪彬

INTERNATIONAL GCSE

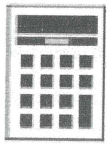
MATHEMATICS

Extension Tier

Tuesday 23 September 2024 Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have:
- a pencil and a ruler
- a scientific calculator
- the periodic table (enclosed).



Instructions

- Use black ink or black ball-point pen.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- Show all your working.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.
- You are expected to use a scientific calculator where appropriate

For Examiner	
Questions	Mark
1	4
2	3
3	3
4	2
5	3
6	4
7	5
8	0
9	4
10	3
11	3
12	0
13	3
14	6
15	5
16	5
17	2
18	6
19	8
Total:	64

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Q1.

(a) Write 80% as a decimal.

Answer 0.8

(1)

(b) Write 0.7 as a fraction.

Answer $\frac{7}{10}$

(1)

(c) Write $\frac{3}{4}$ as a decimal.

Answer 0.75

(1)

(d) Write 80%, 0.7 and $\frac{3}{4}$ in order with the smallest first.

0.8 0.7 0.75

0.7 < 0.75 < 0.8

Answer 0.7, $\frac{3}{4}$, 80%

(1)

(Total 4 marks)

Q2.

Work out $\frac{3}{4} + \frac{7}{11}$

Give your answer as a mixed number.

You **must** show your working.

$\frac{3}{4} = \frac{33}{44}$

$\frac{7}{11} = \frac{28}{44}$

$\frac{33}{44} + \frac{28}{44} = \frac{61}{44} = 1\frac{17}{44}$

Answer $1\frac{17}{44}$

(Total 3 marks)

Q3.

(a) Circle $\frac{41}{330}$ written as a recurring decimal.

$0.1\dot{2}4\dot{}$ 0.124242424 $0.12\dot{4}$ $0.\dot{1}2\dot{4}$

(1)

(b) Convert $0.2\dot{1}\dot{6}$ to a fraction in its simplest form.

You **must** show your working.

Set 0.216 as x.

~~$x = 0.2 = 0.0\dot{1}6$~~

$100x = 21.6\dot{1}6$

$100x - x = 21.4$

$99x = 21.4$

$x = \frac{107}{495}$

Answer $\frac{107}{495}$

(3)

(Total 4 marks)

Q4.

Simplify $\frac{4 \times 2^5}{8^{\frac{1}{3}}}$

Give your answer in the form 2^k where k is an integer.

You **must** show your working.

$\frac{4 \times 2^5}{8^{1/3}} = 2^2 \times 2^5 \times 2^{-1}$

$= 2^6$

$k = 6$

Answer 6

(Total 3 marks)

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Q5.

By rationalising the denominator, show that $\frac{21+\sqrt{7}}{3-\sqrt{7}}$ simplifies to $a+b\sqrt{7}$ where a and b are integers.

You **must** show your working.

$\frac{21+\sqrt{7}}{3-\sqrt{7}} = (21+\sqrt{7})(3+\sqrt{7}) / (3-\sqrt{7})(3+\sqrt{7})$

$= (21+\sqrt{7})(3+\sqrt{7}) / 9-7$

$= (70+24\sqrt{7}) / 2$

$= 35+12\sqrt{7}$.

~~$a=45, b=12$~~ $a=35, b=12$

(Total 3 marks)

Q6.

(a) Write 126 as a product of prime factors.

2, 3, ~~6~~, ~~7~~

~~$2\times3\times6\times2$~~ $2\times3\times3\times7=126$

Answer 2, 3, 3, 7

(2)

(b) Work out the Highest Common Factor (HCF) of 72 and 126.

18.
72 ÷ 18 = 4
126 ÷ 18 = 7.

6 | 72 1 26
3 | 12 21
4 7

HCF = 3 × 6 = ~~18~~.

Answer 18

(2)

(Total 4 marks)

Q7.

$\frac{a}{b}$ and $\frac{b}{c}$ are two **proper** fractions.

Each fraction is in its simplest form.

The lowest common denominator of the two fractions is 35

The sum of $\frac{a}{b}$ and $\frac{b}{c}$ is also a **proper** fraction.

Work out the value of $\frac{a}{b} + \frac{b}{c}$

0

1. a<b, b<c

2. b · c = 35 = 5 × 7

∴ b = 5, c = 7

35 = 1 × 35 = 5 × 7

b = 5 ✗ c = 7, b = 5

b = 7 ✗ $\frac{a}{5} + \frac{5}{7}$

c = 5 ✗

c = 7 ✓

Answer _____

(Total 2 marks)

Q8.

How many factors does the number 2^{20} have?

Circle your answer.

20 21 22 40

(Total 1 mark)

Q9.

(a) Factorise $2x^2 - x - 3$

$2x \times -3$

$2x \times 1$

$(2x-3)(x+1)$

Answer $(2x-3)(x+1)$

(2)

(b) Hence, simplify $\frac{2x^2 - x - 3}{4x^2 - 9}$

$2x^2 - x - 3 / 4x^2 - 9 = (2x-3)(x+1)/(2x+3)(2x-3)$

$= (x+1)/(2x+3)$

Answer $x+1/2x+3$

(2)

(Total 4 marks)

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Q10.

The expression $\frac{x^2-9}{x^2-bx-15}$ simplifies to $\frac{x-3}{x-5}$

Work out the value of b.

$x^2-bx-15 = x-5$

$(x^2-9) = (x+3)(x-3)$.

$\therefore x^2-bx-15 = (x+3)(x-5)$

$= x^2-2x-15$.

$\therefore b=2$.

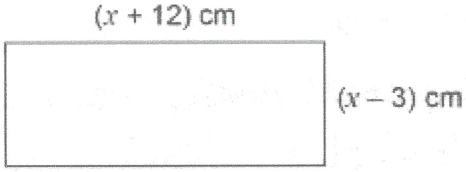
$b =$ 2

(Total 3 marks)

Q11.

A rectangle has length $(x + 12)$ cm and width $(x - 3)$ cm

Not drawn accurately



The length is twice the width.

Work out the value of x.

$(x+12) = 2(x-3)$

$x+12 = 2x-6$

$x = 18$

Answer 18

(Total 3 marks)

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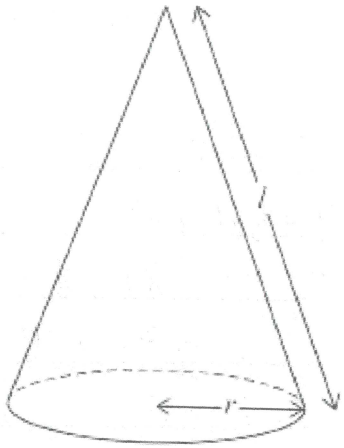
Q12.

A cone has radius r and slant height l.



Total surface area of a cone = $\pi r(r + l)$

The total surface area of the cone is **exactly** 152π cm^2

The radius of the cone is 4.5 cm to the nearest 0.5 cm

Work out the lower bound of l.

You **must** show your working.

The lowest possible number of $r = 4.45$

$4.45\pi(4.45 + l) = 152\pi$

$19.8025\pi + 4.45l\pi = 152\pi$

$4.45l\pi = 132.198\pi$ $\quad l = 29.7074$

The lower bound of $l = 29.7074$

$4.25 \leq r < 4.75$ ±

$l = 152/r - r$

~~$l = 27$~~

Answer 29.7074 cm

(Total 4 marks)

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Q13.

70 biscuits are put into an empty tin.
Each biscuit has a mass of 12 g to the nearest gram.
The total mass of the tin and biscuits is 950 g to the nearest gram.
Work out the upper bound of the mass of the empty tin.
You **must** show your working.

Lowest possible of each biscuit = 11.5 g.
Highest possible of Total mass = 950.49 g
70 x 11.5 = 805g.
950.49 - 805 = 145.49
The Upper bound of the mass of the empty tin = 145.49

Answer 145.49 g

(Total 3 mark)

Q14.

Here are two functions.

$f(x) = 3x - 1$ $g(x) = x^2 + 3x + 2$

(a) Work out an expression for $f^{-1}(x)$

$y = 3x - 1$
$x = 3y - 1$
$-3y = -x - 1$
$-y = \frac{-x-1}{3}$ $f^{-1}(x) = \frac{x+1}{3}$
$y = \frac{x+1}{3}$

Answer $\frac{x+1}{3}$

(2)

(b) Work out the value of $g(-3)$

$g(-3) = 9 - 9 + 2$

$g(-3) = 2$

Answer 2

(1)

(c) Work out an expression for $gf(x)$

Simplify your answer.

$gf(x) = (3x-1)^2 + 3(3x-1) + 2$

$gf(x) = 9x^2 - 6x + 1 + 9x - 3 + 2$

$gf(x) = 9x^2 + 3x$

Answer $9x^2 + 3x$

(3)

(Total 6 marks)

Q15.

Simplify fully $\frac{9x^2 - 1}{3x^2 + 2x - 1} \div \frac{3x + 1}{x - 2}$

Original Sheet $= (3x+1)(3x-1) / (3x-1)(x+1) \times x-2 / 3x+1$

$= (3x+1)/(x+1) \times (x-2)/(3x+1)$

$= x-2/x+1$

Answer $\frac{x-2}{x+1}$

(Total 5 marks)

Q17.

By completing the square, solve $2x^2 - 20x + c = 0$

Give your answers in surd form **in terms of** c.

You **must** show your working.

$2(x^2 - 10x + c/2) = 0$

$x^2 - 10x + c/2 = 0$

$(x^2 - 10x + 25) - 25 + c/2 = 0$

$(x-5)^2 - 25 + c/2 = 0$

Answer $(x-5)^2 - 25 + c/2 = 0$

(Total 3 marks)

Q18.

It is given that $f(x) = 2x^2 - 16x + 38$

(a) Express $f(x)$ in the form $a(x - b)^2 + c$, where a, b and c are positive integers.

$f(x) = 2(x^2 - 8x + 19)$

$f(x) = 2(x^2 - 8x + 16) + 38 - 32$

$f(x) = 2(x-4)^2 + 6$

Answer $2(x-4)^2 + 6$

(3)

Q16.

$f(x) = x^2 - x - 1$

$g(x) = 1 + \frac{1}{x} \quad x \neq 0$

(a) Write $f(x)$ in the form $(x-a)^2 - b$ where a and b are constants.

$f(x) = (x^2 - x + 1/4) - 1 - 1/4$

$f(x) = (x - 1/2)^2 - 1 1/4$

$a = 1/2$, ~~$b = 1 1/4$~~ $b = 1 1/4$

Answer $(x - 1/2)^2 - 1 1/4$

(2)

(b) State the range of $f(x)$

~~$0 \leq f(x) \leq \infty$~~ $-1 \leq f(x) < \infty$

Answer $f(x) \geq -1$

(1)

(c) Work out an expression for $g^{-1}(x)$

$g(x) = y = 1 + 1/x$

$g^{-1}(x) = x = 1 + 1/y$

$1/y = x - 1$

$y = \frac{1}{x-1}$

$g^{-1}(x) = \frac{1}{x-1}$

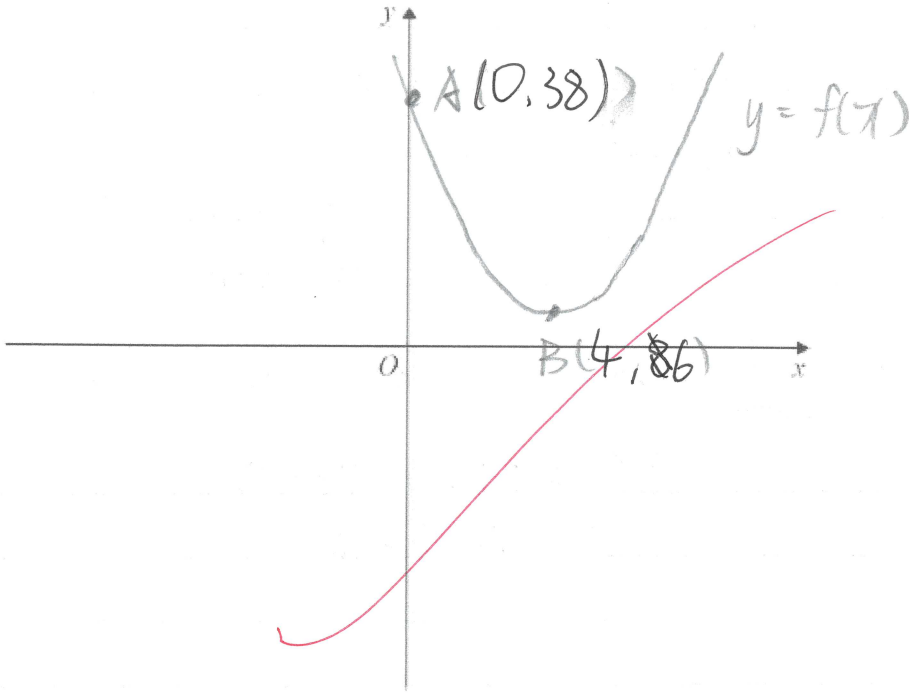
Answer $\frac{1}{x-1}$

(3)

(Total 6 marks)

(b) The curve C with equation $y = f(x)$ crosses the y-axis at the point A and has a vertex at B.

Sketch the graph of C, showing the coordinates of A and B.



Answer A(0,38) B(4,6)

(3)

(Total 6 marks)

8

Q19.

(a) Factorise $x^2 - 4x - 12$.

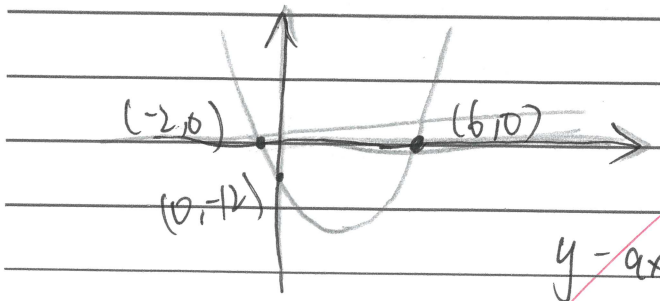
$x \times 2$ $(x+2)(x-6)$

$x \times -6$

Answer $(x+2)(x-6)$

(1)

(b) Sketch the graph with equation $y = x^2 - 4x - 12$, stating the values where the curve crosses the coordinate axes.



y-axis (0, -12)
x-axis (6, 0) (-2, 0)

(4)

(c) (i) Express $x^2 - 4x - 12$ in the form $(x - p)^2 - q$, where p and q are positive integers.

$(x^2 - 4x + 4) - 4 - 12$

$(x-2)^2 - 16$

Answer $(x-2)^2 - 16$

(2)

(ii) Hence find the minimum value of $x^2 - 4x - 12$.

When $x = 2$, has minimum value of -16.

Answer -16

(1)

(Total 8 marks)