2. Types of Transformation

4 Types:

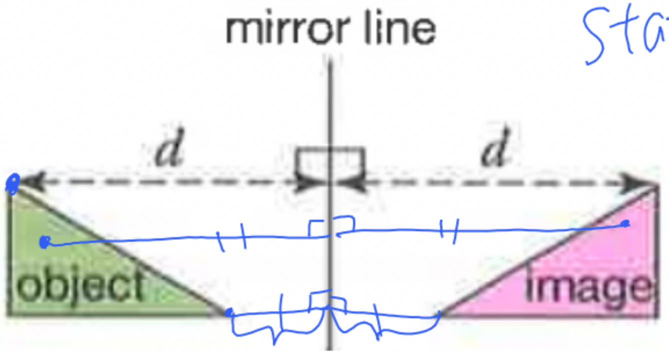
To describe a	give
Reflection	The position of the mirror line
Rotation	The angle of rotation The direction (clockwise or anti-clockwise) The centre of rotation
Translation	The vector or the distance and direction
Enlargement	The scale factor The centre of enlargement

(1) Reflection

翻转 镜像线

- **Reflection** flips a shape in a **mirror line** (also called a line of reflection) so that each point is the same distance from the mirror line as its reflected point.
- The shapes are congruent.

State what's the mirror line



G21

Core content	Extension content
describe and transform 2D shapes using single rotations, reflections, translations, or enlargements by a positive scale factor and distinguish properties that are preserved under particular transformations	including combined transformations and enlargements by fractional and negative scale factors

Notes: translations will be specified by a vector.

>0. <0.

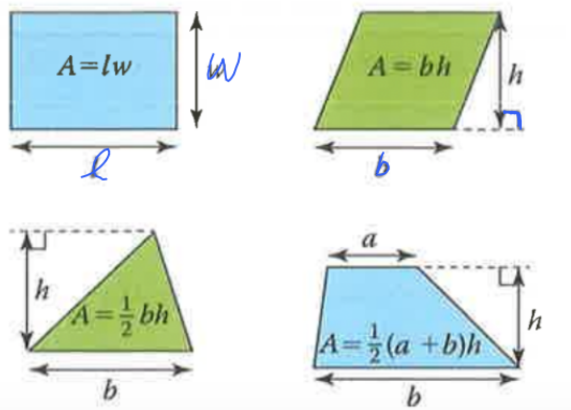
Study Goals:

- Types of transformation + distinguish
  - Rotation-center of rotation
  - Reflection-mirror line
  - Translation-vector
  - Enlargement-scale factor (SF), center of enlargement
- Combined transformations

1. Area of a 2D shape

In the metric system, area is measured in **mm²**, **cm²**, **m²** or **km²**

- Area of a rectangle = length × width
- Area of a parallelogram = base × perpendicular height
- Area of a triangle = $\frac{1}{2}$ base × perpendicular height
- Area of a trapezium = $\frac{1}{2}$ sum of the parallel sides × perpendicular height



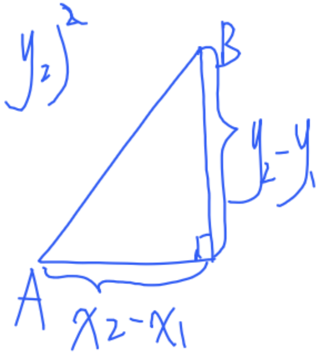
Question Type 1: Describe a reflection

$AB = \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$

1. **Pair up the points.**
2. **Identify the midpoints.**
3. **Join the midpoints.**
4. **State the equation of the line.**

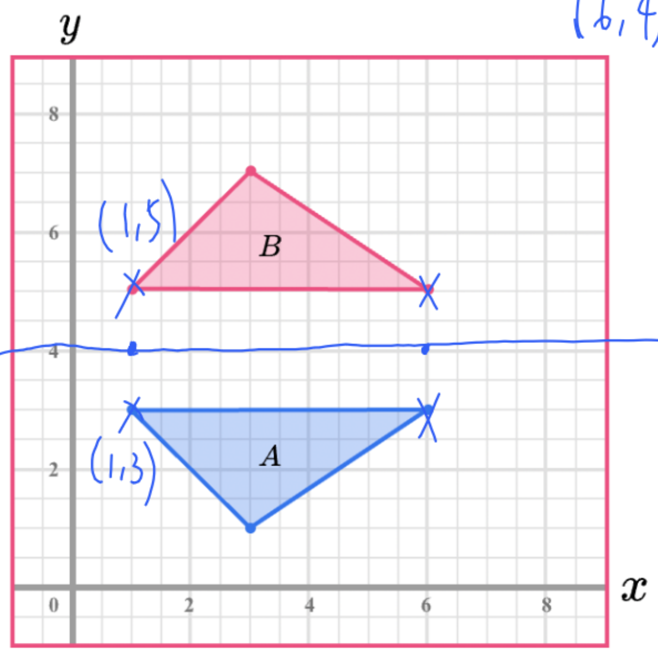
$A(x_1, y_1)$. $B(x_2, y_2)$.

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Describe the transformation of Shape A to Shape B

$(1,4)$
$(6,4)$



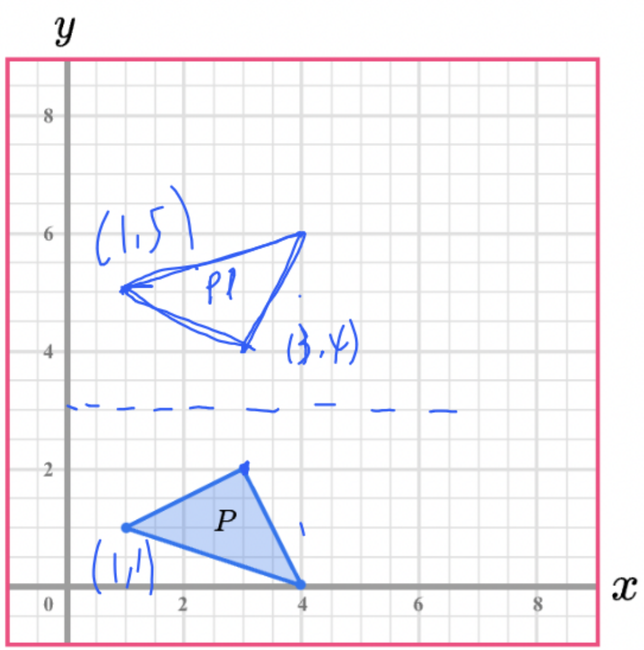
$y = 4$

Question Type 2: reflect a 2D shape

In order to reflect a shape on a coordinate grid:

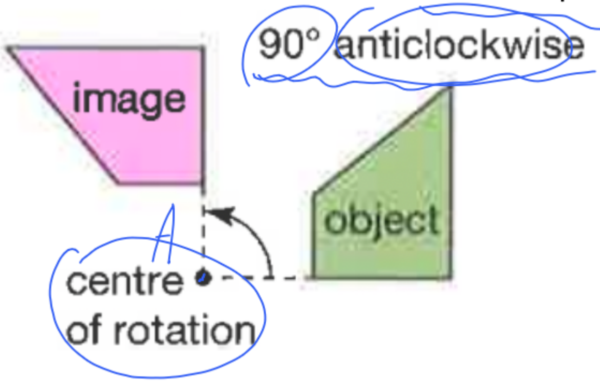
1. **Draw the mirror line.**
2. **Reflect the first point.**
3. **Reflect the other points.**
4. **Finish the diagram.**

Reflect Triangle P in the line $y = 3$: mirror line



(2) Rotation

- Rotations are transformations that turn a shape around a fixed point.



To rotate a shape we need:

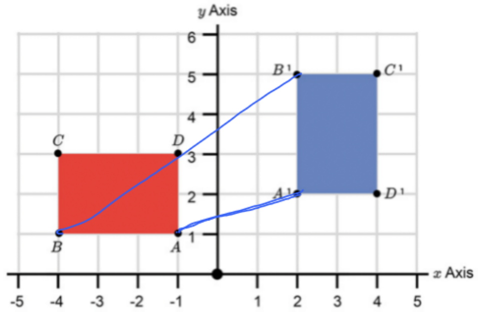
- a **centre of rotation**
- an **angle of rotation** (given in degrees)
- a **direction of rotation** – either clockwise or anti-clockwise. (Anti-clockwise direction is sometimes known as counterclockwise direction).

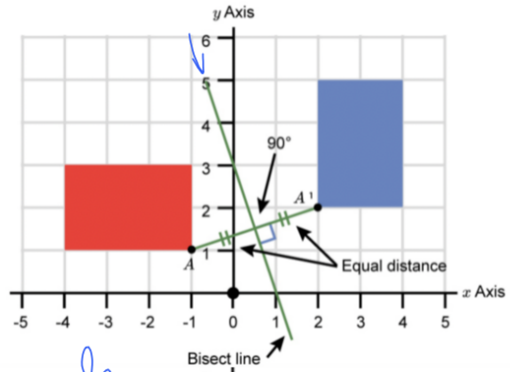
Clockwise **Anti-clockwise**

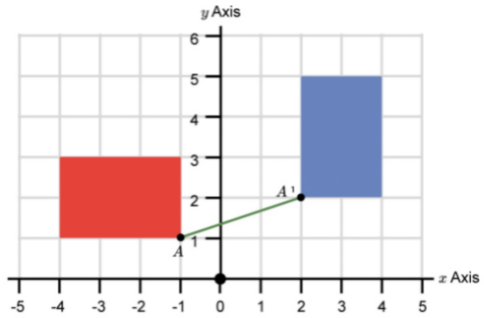
Q: How to find the center of rotation

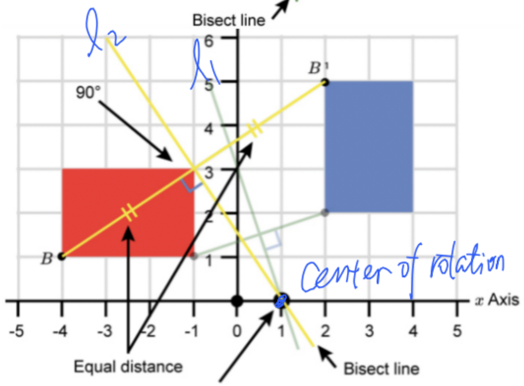
垂直平分线

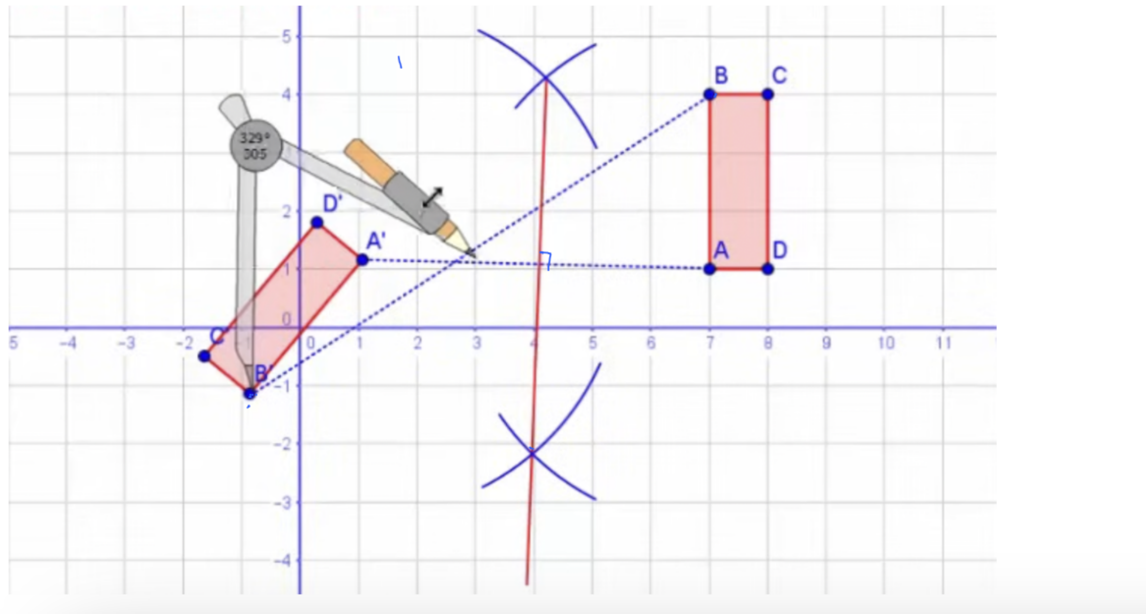
- Step 1: connect the corresponding points
- Step 2: draw the perpendicular bisector to this line
- Step 3: try another pair of points, Repeat the first two steps
- Step 4: The intersection of the two vertical bisectors is ours **center of rotation**


y Axis
x Axis
B'
C'
A'
D'
C
D
B
A

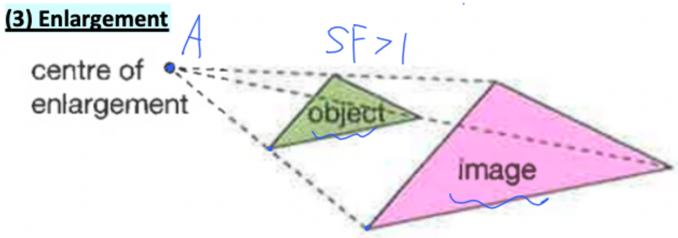

y Axis
x Axis
90°
A'
A
Equal distance
Bisect line


y Axis
x Axis
A'
A


l2
l1
90°
B'
B
Center of rotation
x Axis
Equal distance
Bisect line

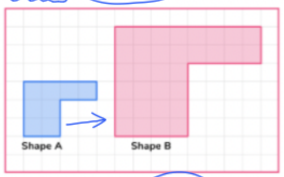

B
C
A
D
D'
A'
C
B'
329°
305

(3) Enlargement

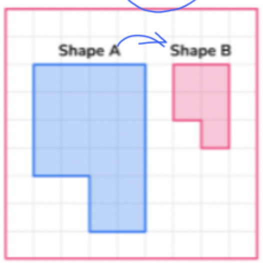


In an enlargement the distance from the centre of enlargement to every other point is multiplied by a **scale factor** (SF).

- $\boldsymbol{SF > 1}$: **enlarges**



- $\boldsymbol{0 < SF < 1}$: **smaller**



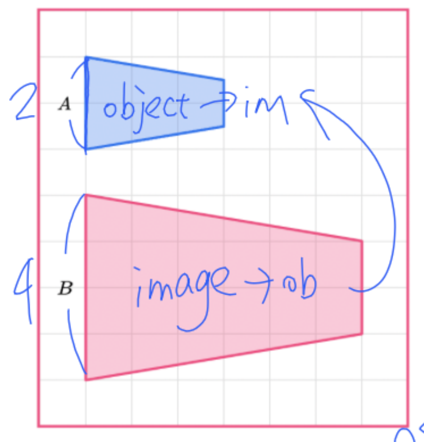
In order to calculate a scale factor:

1. Choose a pair of corresponding sides.

2. Divide the length of the enlarged shape by the length of the original shape.

3. Write down the scale factor.

Calculate the scale factor for the enlargement of Shape A to Shape B:



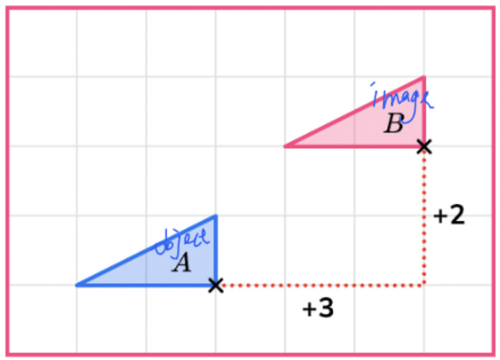
$\frac{4}{2} = 2 = SF$
> 1

$0 < SF = \frac{1}{2} < 1$

(4) Translation

- In a **translation**, *all* points move by the same distance in the same direction.
- The shapes are congruent.

E.g.



$\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ left, down

- Translations involve a move in a horizontal direction and a move in a vertical direction.

==> We use a **column vector** to help record the movement.

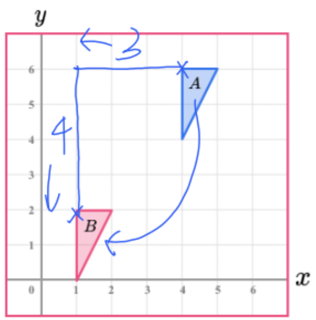
Shape A has been translated to shape B by the column vector

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ *is* $\begin{pmatrix} 3\ right \\ 2\ up \end{pmatrix}$

Question Type 1: Describe a translation

1. Pair up the points.
2. Work out the horizontal movement.
3. Work out the vertical movement.
4. State the column vector.

Describe the translation of shape A to shape B



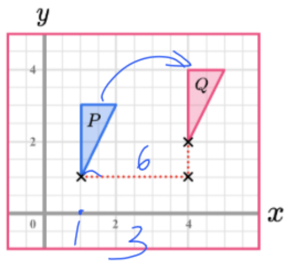
$\begin{pmatrix}-3\\-4\end{pmatrix}$

B → A

$\begin{pmatrix}3\\4\end{pmatrix}$

**Common misconceptions:

- **Check the *scale* of the coordinate grid**
  What is the column vector for the translation of shape P to shape Q?



If you count the squares, the column vector is: $\begin{pmatrix}6\\2\end{pmatrix}$
But if considering the scale on the axes, the correct vector is: $\begin{pmatrix}3\\1\end{pmatrix}$

- **Interpreting the column vector**

- **Interpreting the column vector**
  - Remember, the top number is for horizontal movement. (positive: moves right)
  - The bottom number is for vertical movement. (positive: moves upwards)
- **Object and image**
  P->Q or Q->P

Question Type 2: translate a 2D shape

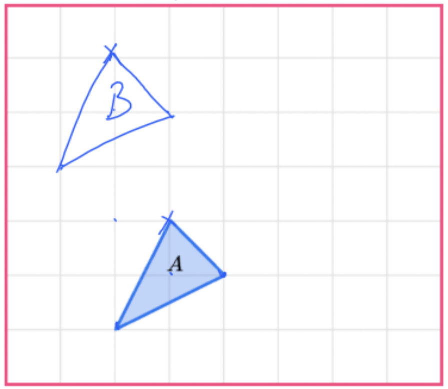
1. Choose a starting point on the shape.
2. Move it across. right/left
3. Move it up or down.
4. Complete the rest of the shape.

Translate shape A by the column vector and label the image B

$$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

left 1

up 3



Example:

EXAMPLE

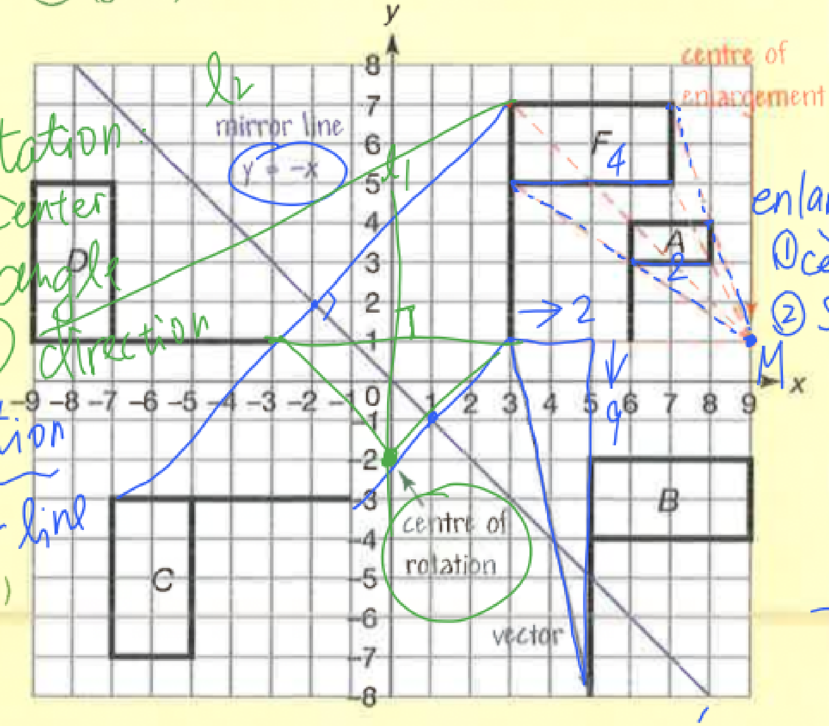
Write a full description of the transformation that maps the flag *F* onto

object

a *A* **b** *B* **c** *C* **d** *D*.

a (2) *A* has changed size, it is half as tall and half as wide as *F* ⇒ enlargement.
Join corresponding points to find the centre of the enlargement.

(1) (given)



(3) Enlargement, scale factor $\frac{1}{2}$, centre (9, 1)

rotation
① center
② angle
③ direction

enlargement
① center of ... M
② SF = $\frac{2}{4} = \frac{1}{2}$

b (2) *B* is in the same orientation but a different position ⇒ translation.
F moves right by 2 squares and down by 9 squares.

(3) Translation by vector $\begin{pmatrix} 2 \\ -9 \end{pmatrix}$

reflection
mirror line

$\begin{pmatrix} 2 \\ -9 \end{pmatrix}$

Translation vector

c (2) *C* is 'flipped' ⇒ reflection.
Points like (−2, 2), (−1, 1) and (1, −1) lie halfway beween *F* and *C*.

(3) Reflection in mirror line y = −x

d (2) *D* is turned, 90° anti-clockwise ⇒ rotation.
Use tracing paper to find the centre of rotation. (You will use perpendicular bisectors later.)

(3) Rotation 90° anti-clockwise about (0, −2).

In Summary:

1.

- In a reflection, rotation or translation the image and object shapes are **congruent**.
- In an enlargement the image and object shapes are **similar**.

2.

RECAP

To identify the type of transformation compare the object and the image.

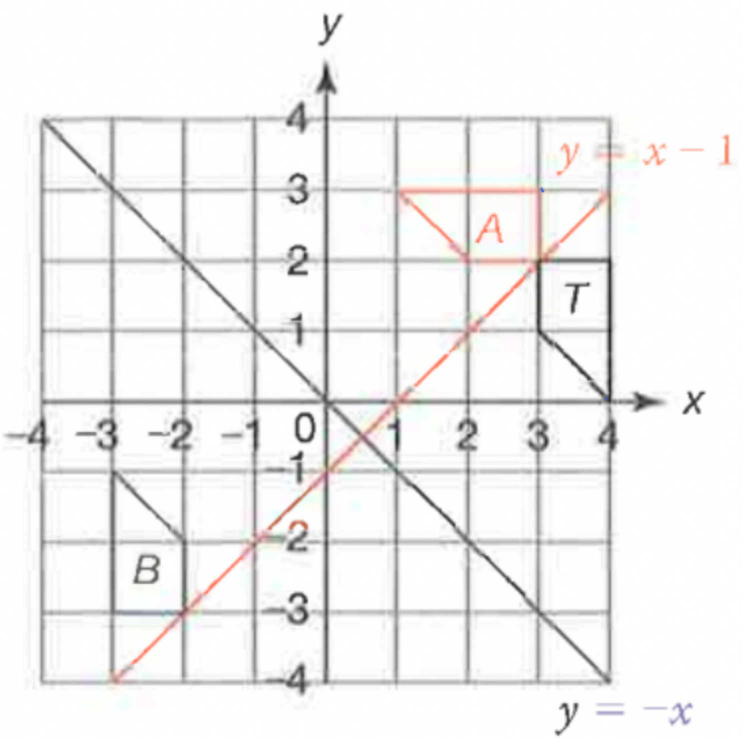
- Congruent shapes, same orientation ⇒ translation vector $\begin{pmatrix} 20 \\ \cancel{30} \end{pmatrix}$
  Give vector (or distance and direction)
- Congruent shapes, image 'flipped' ⇒ reflection
  Give mirror line
- Congruent shapes, image 'turned' ⇒ rotation
  Give, centre ① , angle ② and direction ③ clockwise
- Similar shapes ⇒ enlargement
  - image same orientation and enlarged scale factor > 1 enlarge
  - image same orientation and reduced scale factor between 0 and 1
  - image inverted scale factor negative

  Give centre ① and scale factor ②

3. Combined Transformations

Two or more transformations may be combined.

The result may be equivalent to a single transformation.



▲ Reflection of shape T in $y = x - 1$ followed by reflection in $y = -x$.

T, A and B are congruent.
Overall T has rotated 180° about the point $\left(\frac{1}{2}, -\frac{1}{2}\right)$.