G11

Core content	Extension content
interpret plans and elevations of 3D shapes construct and interpret plans and elevations of 3D shapes	

G15

Core content	Extension content
know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of 3D shapes using $V = Ah$ where A is the constant cross sectional area and h is the height/length	

G16

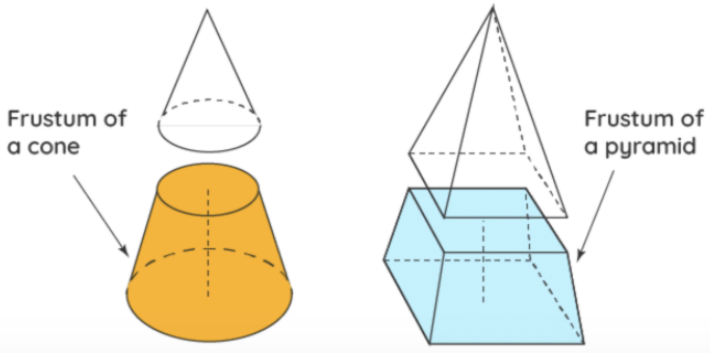
Core content	Extension content
know and use the formulae: circumference of a circle = $2\pi r = \pi d$ area of a circle = πr^2 calculate perimeters and areas of 2D shapes, including composite shapes	surface area and volume of spheres, pyramids, cones and composite solids including composite shapes and frustums of pyramids and cones

Notes: solutions in terms of π may be asked for.

3D (types volumes. Surface. areas

Study Goals:

- Calculate the volume of cuboids and prisms
- Calculate the surface area and volumes of spheres, pyramids, cones
- Calculate the volumes of composite shapes (**frustums** of pyramids, cones)

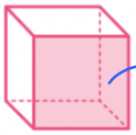
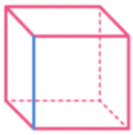


Vocabularies

Name	Translate	Name	Translate
prisms	棱镜	front /side elevation	正面/侧面视图
sphere	球体	plan	平面图
pyramid	锥体	face	面
cone	圆锥体	edge	边
Composite shape	复合形状	vertex	顶点
solid	立体图形	frustums	锥台
cross-section	横截面	polyhedron	多面体
Cylinder /ˈsɪlɪndə(r)/	圆柱体	lateral	侧面的
net	展开图	apex	顶点

1. **Definition**

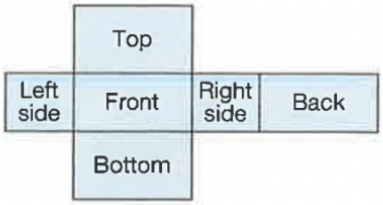
- 3D shapes are **solid shapes** that have **3 dimensions**: height (h), width (w) and depth (d).
- Faces, edges and vertices:

Face A flat 2D shape surrounded by edges and vertices.	
Edge A straight line where two faces meet and connects two vertices	
Vertex A point where two or more edges meet (like a corner)	

- 3D shape **nets**

  - A **net** is a 2D shape that can be folded to make a 3D shape.

Imagine cutting along some of the edges of the cuboid and opening it out to give this net.



2. Types of 3D shapes

They can be categorized as either **polyhedra**, or **non-polyhedra (curved)**.

Polyhedra	Non-polyhedra
(3D shapes with all flat polygonal faces)	(3D shapes which have a curved surface)

Common polyhedron:

- Prism: It has identical polygon ends and flat parallelogram sides
- Cube: It has 6 square faces, 8 vertices and 12 edges
- Cuboid: It has 6 rectangular faces, 8 vertices and 12 edges
- Pyramid: It has a polygon base, straight edges, flat faces and one vertex

Common curved solid (non-polyhedron):

- Sphere: a round shape, having all the points on the surface equidistant from center 球体
- Cone: It has a circular base and a single vertex
- Cylinder: It has parallel circular bases, connected through curved surface 圆柱

We'll look at each figure individually:

- **Definition**
- **Volume**
- **Surface area**
- **Nets**

Volume is measured in **mm**^{3}, **cm**^{3}, **m**^{3} or **km**^{3}.
$1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3$
Liquids are usually measured in **litres** or **millilitres**.
$1\,\text{m}^3 = 1000$ litres, 1 litre $= 1000\,\text{cm}^3$ and $1\,\text{ml} = 1\,\text{cm}^3$

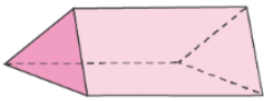
Common polyhedron:

3. **Prism 棱镜**

Definition

- A prism has the same cross-section throughout its length.上下底面平行且全等
- prisms are named according to the shape of their cross-sections
- Cubes, cuboids are all prisms.

Square Prism



Triangular Prism

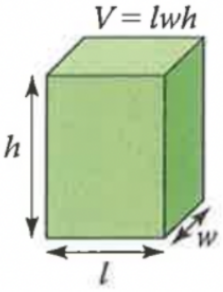
Pentagonal Prism

Volume & surface area

Volume of a prism = area of cross-section × length

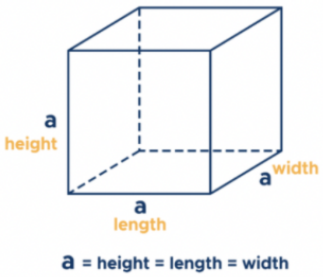
Surface area of a prism = twice the base area + the lateral surface area

4. Cube and cuboid



▲ Cuboid

Surface area = $lw + lh + wh$



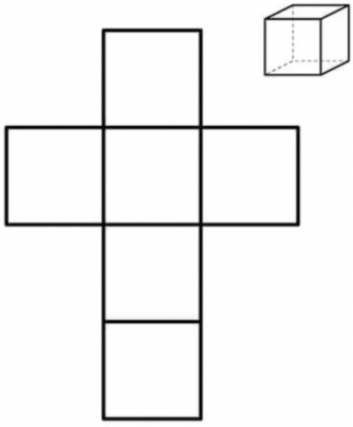
a = height = length = width

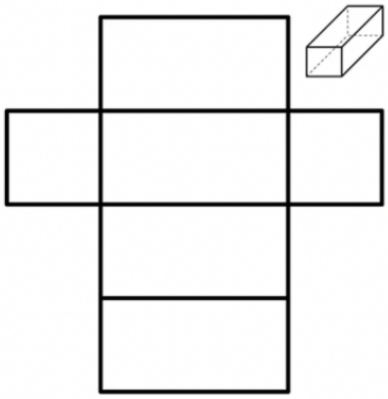
Volume = a^3

Surface area = $6a^2$

Cube	• 6 square faces • 8 vertices • 12 edges
Cuboid	• 6 rectangular faces • 8 vertices • 12 edges

Nets

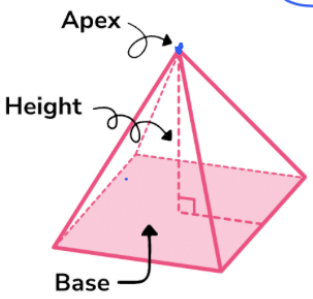


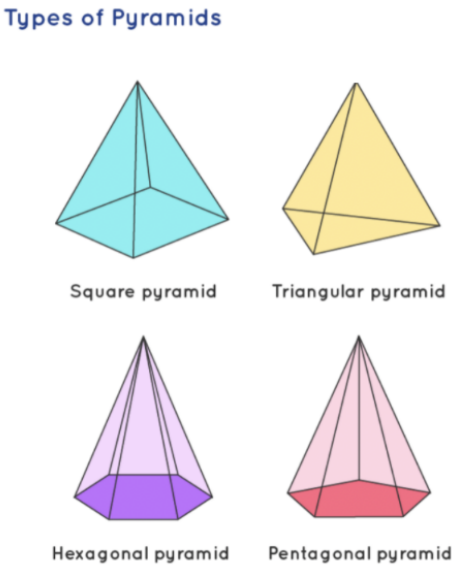


5. Pyramids 锥体

Definition

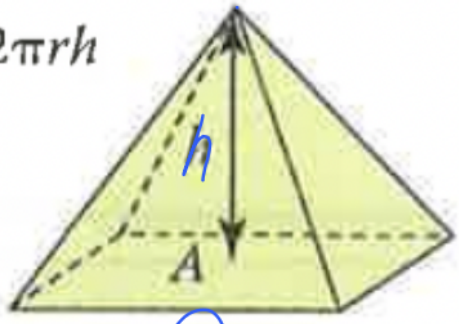
- The base of a pyramid is a polygon. The other faces are triangles
- The apex is the convergent point to which all the slanted edges of the pyramid meet.
- **If a pyramid has an n-sided base, then it has n+1 faces, n+1 vertices, and 2n edges**





Volume

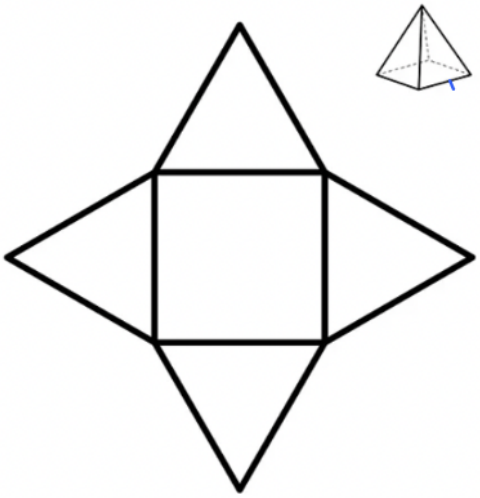
$= 2\pi rh$



$V = \frac{1}{3}Ah$

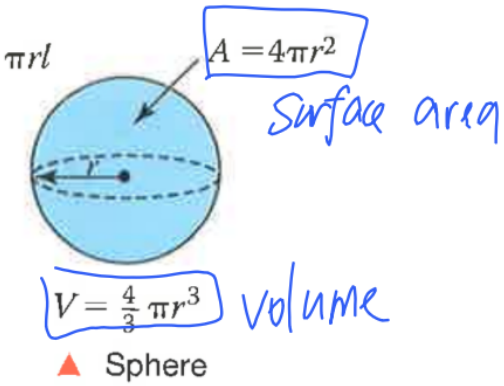
▲ Pyramid

Net square pyramid



Common curved solid (non-polyhedron):

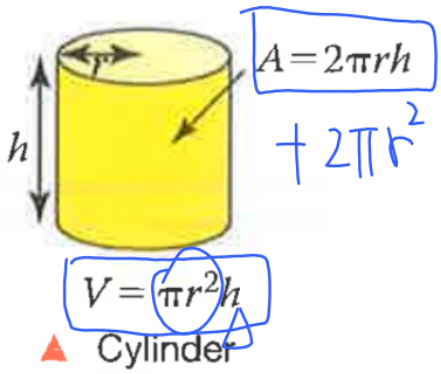
6. Sphere 球体



▲ Sphere

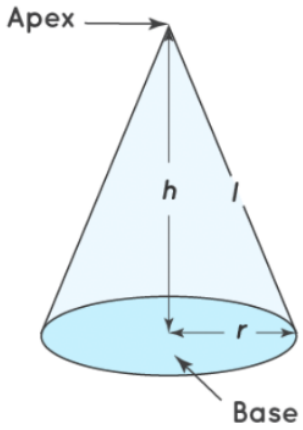
7. Cylinder 圆柱体

- two circular faces, one at the top and one at the bottom
- one curved surface
- height and a radius



▲ Cylinder

8. Cone 圆锥

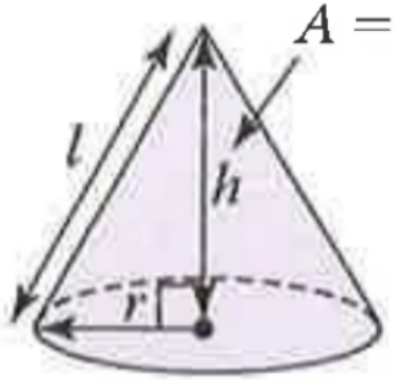


h = Height of the cone

r = Radius of the cone

l = Slant height of the cone

$l = \sqrt{r^2 + h^2}$



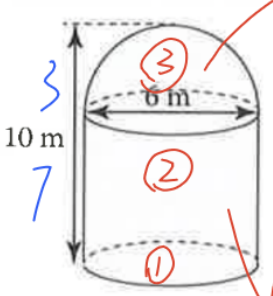
$A = \pi rl + \pi r^2$

$V = \frac{1}{3}\pi r^2 h$

▲ Cone

Example:

Find the surface area and volume of this solid.



$V = \frac{1}{2}\frac{4}{3}\pi r^3$ $V = \frac{2}{3}\pi r^3 + \pi r^2 h$ $-81\pi / 254\,m^3$

$V = \frac{4}{3}\pi r^3$

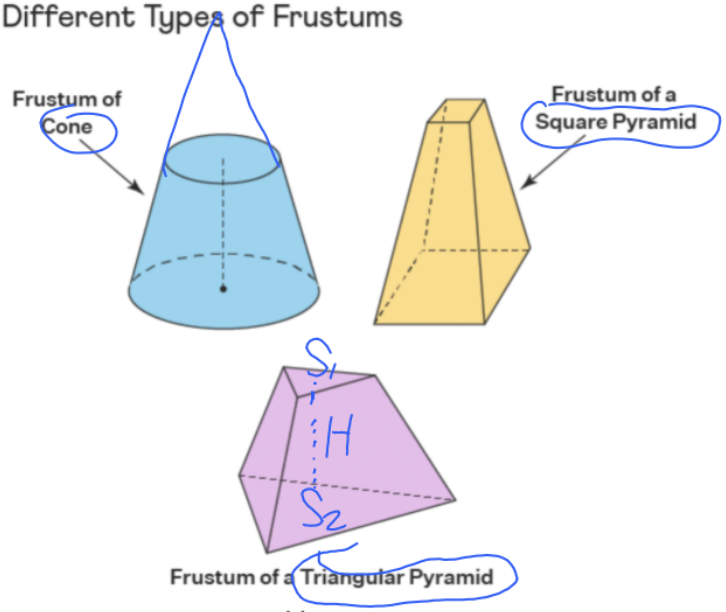
$3^2\pi \cdot 7 + \frac{4}{3}\pi \cdot 3^3 \cdot \frac{1}{2} = 203.64\,m^3$

$V = \pi r^2 \cdot h$

$2\pi r^2 + 2\pi r \cdot h + \pi r^2$

9. Frustum

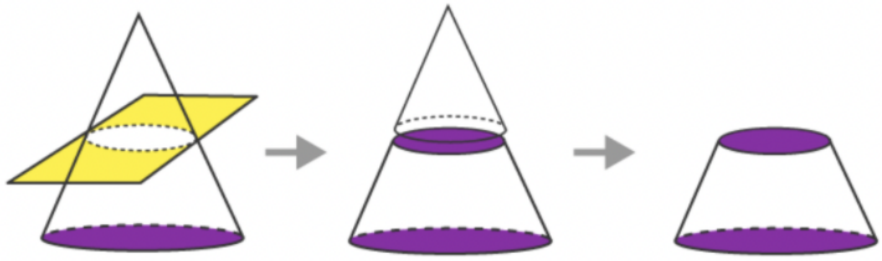
Different Types of Frustums



Volume of frustum, $V = \frac{H}{3}(S_1 + S_2 + \sqrt{S_1 S_2})$, where

- H = Height of the frustum (the distance between the centers of two bases of the frustum)
- S_1 = Area of one base of the frustum
- S_2 = Area of the other base of the frustum

Frustum of a cone

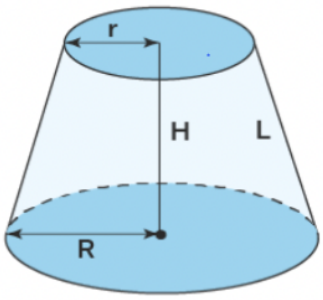


Plane parallel to base cuts the cone

Cone as two separate parts

Frustum of a cone

Frustum of cone



$V = \frac{H}{3}\left(S_1 + S_2 + \sqrt{S_1 S_2}\right) = \frac{\pi h}{3}\left(R^2 + r^2 + rR\right)$

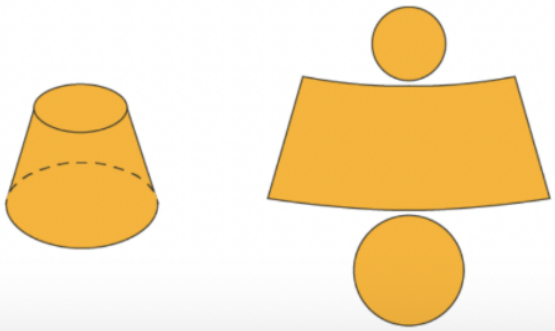
$S_1 = \pi r^2 \quad S_2 = \pi R^2 \quad \sqrt{\pi r^2 \cdot \pi R^2} = \pi rR$

Frustum of a Cone Formulas

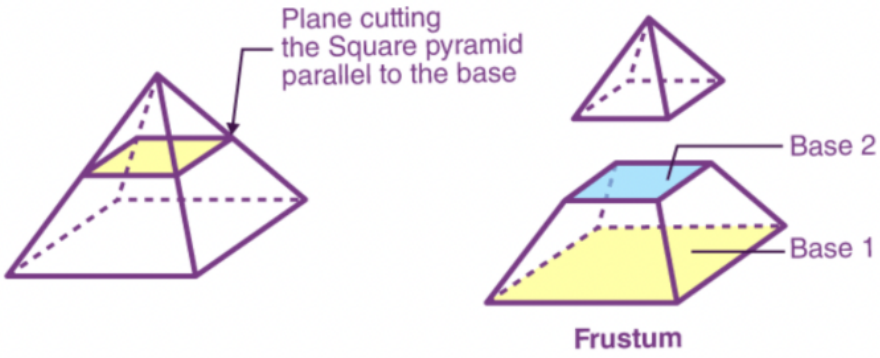
Consider height to be 'h', slant height to be 'l', 'r' and 'R' be the two radii of circular bases of the frustum respectively. Then the formula related to the frustum of a cone is tabulated below:

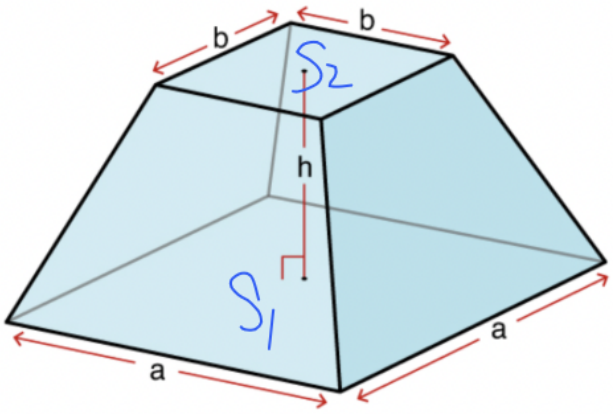
Frustum of a Cone	Formula
Slant Height of Frustum of a Cone	$l = \sqrt{h^2 + (R - r)^2}$
Curved Surface Area (CSA) or Lateral Surface Area (LSA)	CSA or LSA = $\pi(R + r)l$
Total Surface Area (TSA)	TSA = $\pi[(R + r)l + r^2 + R^2]$
Volume	$V = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

Net



Frustum of pyramid

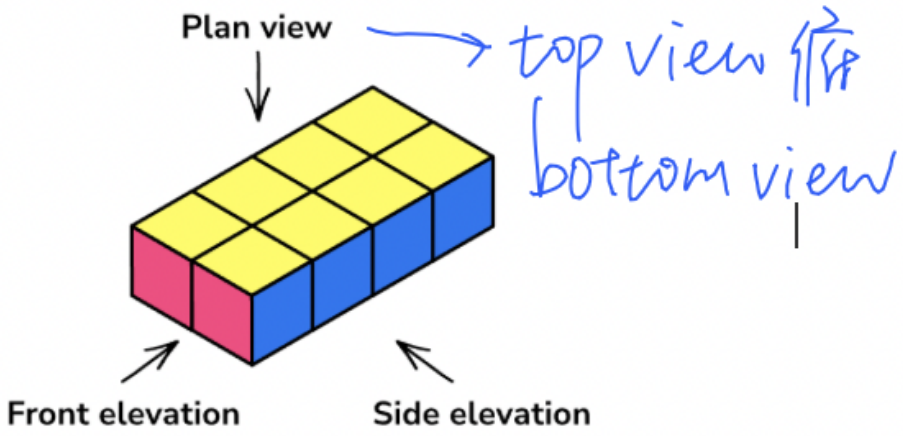




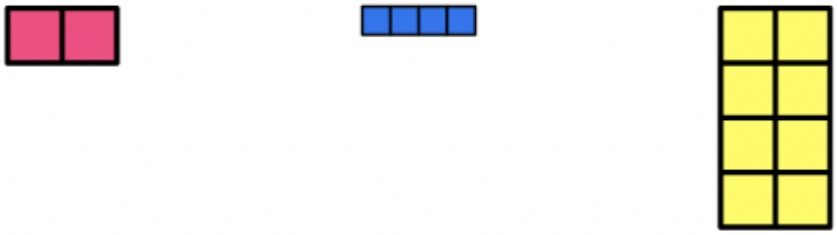
$$V = \frac{1}{3} h (a^2 + b^2 + ab)$$

10. Plans and Elevations

- 3D shapes can be represented by **plans** and **elevations**.



Front elevation　　Side elevation　　Plan view



Front elevation：正视图

Side elevation：侧视图

Plan view：平面视图

--top view：俯视图

--bottom view：底视图